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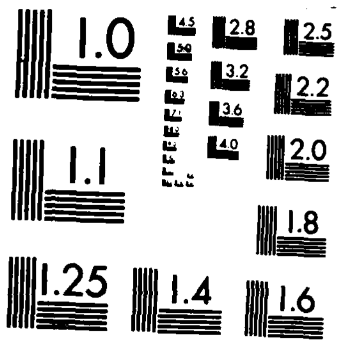

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PERFORMANCE OF DS/SSMA COMMUNICATIONS IN IMPULSIVE CHANNELS

Behnaam Aazhang

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

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PERFORMANCE OF DS/SSMA COMMUNICATIONS IN IMPULSIVE CHANNELS

BY

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TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The objective of this thesis is to analyze the performance of asynchronous binary PSK direct-sequence spread-spectrum multiple-access (DS/SSMA) communications over an additive impulsive noise channel. In recent years, there have been a number of studies of such communication systems with the additive white Gaussian assumption for channel noise. In particular, many researchers have devoted their efforts to finding efficient methods of obtaining approximations and bounds for the average bit-error-probability of the conventional correlation receiver within this model [11-12], [26-27], and [43-44]. Pursley [26] developed an approximation for the average error probability based on the average signal-to-noise ratio. Using moment-space ideas, Yao [44] developed upper and lower bounds on the error probability. Later, Wu and Neuhoff [43] used series expansions and Gauss quadrature rules to obtain estimates for the performance of the conventional linear correlation receiver. Recent contributions to this problem include the methods of approximations given by Geraniotis and Pursley in [11] and [12], which are based on series expansions and the integration of the characteristic function, respectively. Each of these methods has its advantages and disadvantages, and the choice of method for a given application ultimately depends on the system parameters and required accuracy. Taken collectively, a number of efficient techniques are available for evaluating the performance of the linear correlation receiver in additive white Gaussian noise in the DS/SSMA environment. Among the many other contributions to the performance analysis of DS/SSMA communications are the studies of Anderson and Wintz and others (see [5]) who have analyzed DS/SSMA systems with a hard limiter in the structure of the receiver, and of Verdu ([41] and [42]) who has considered an optimum detection algorithm that yields a minimum error probability DS/SSMA receiver.

One common assumption among the contributions referenced above is that of additive white Gaussian noise. Although this assumption is quite appropriate for many applications, it is well-known that many noise environments arising in practice are poorly modeled by Gaussian statistics.

In particular, man-made electromagnetic interference (or noise), and a great deal of natural interference as well, is basically "impulsive" in nature ; i.e., it has a highly structured form, characterized by significant probabilities of large interference levels. This is in contrast to the more "entropic" Gaussian noise processes inherent in transmitting and receiving elements. This impulsive or structured character of the interference can significantly degrade the performance of conventional (linear) demodulation systems, which are designed to operate most effectively against the commonly assumed Gaussian background noise processes. However, by proper system design, this noise structure can often be exploited to yield better performance than would be obtained in the Gaussian case. There have been various efforts over the past two decades in the area of signal detection in impulsive noise, primarily within the context of single-user channels [3], [7-10], [13], [18-23], [36-38], and [40]. Most of these studies are concerned with the problem of detecting the presence of a weak signal in a noisy environment [3], [7], [14], [18], [21-23], [36-38], and [40], and they illustrate that models for the first-and second-order probability distributions of the noise are usually necessary in order to achieve good performance. Since optimum systems for detection in impulsive channels are usually nonlinear, the analysis of such problems is often significantly more difficult than in the conventional Gaussian case.

Even though the two research topics, multi-user communications and data transmission in impulsive noise, have been explored thoroughly, the problem of multiple-access communication in the presence of impulsive noise has not been given much attention (see [8-10]). From the previous work on impulsive channels, one would expect that in multiple-access communication systems operating in impulsive environments, replacing the Gaussian assumption with a more accurate statistical model for the noise and then implementing receivers that take into account the characteristics of that noise would result in better performance than that achieved by the usual linear correlator. However, there is the additional factor of performance against the multiple-access interference which must be considered here, and one might expect that the use of a nonlinear receiver would result in degradation in this aspect of performance. Thus, the overall effect of impulsive and multi-user interferences on linear and nonlinear receivers is unclear.

The purpose of this thesis is to study the effect of the deviations from the Gaussian channel noise assumption in a multi-user environment. In particular, we consider the adaptation of the various methods of performance analysis that have been proposed for either of these problems to the analysis of the combined problem. In this study, the impulsive channel assumption is carried through by modeling the samples of noise after front-end filtering (see also [9-10]). These samples are modeled as independent, identically distributed random variables with first-order probability distribution of various types used to model impulsive noises. The multiple-access capability in the system is achieved by the direct-sequence technique in which the spectrum of the data signal is spread with an assigned "spreading sequence" (see [28-29] and [31-34]). We analyze the performance of the linear and nonlinear DS/SSMA receivers in the described environment with average probability of bit error being the chosen measure of performance.

Our study shows that the linear correlation receiver does not perform as well as the Gaussian model predicts when the non-Gaussian noise has an impulsive nature, even when the signal-to-noise ratio is held constant. For these non-Gaussian impulsive channels, hard-limiting correlation receivers are also examined and are seen to perform with lower error probability than the corresponding linear correlation receiver. However, as one would expect, the linear receiver performs better than the hard-limiting correlation receiver when the channel noise is less structured (e.g., the channel noise is nearly Gaussian). In the hard-limiting correlation receiver example, as the signal-to-noise ratio increases, the hard-limiter apparently is not as effective in separating the two users as the linear correlation receiver is. However, this is outweighed by the improvement against the impulsive noise as long as the channel noise is significant. Alternatively, the soft-limiting correlation receiver, which is expected to perform reasonably well against both impulsive channel noise and and multiple-access interference noise, is studied briefly. Finally, nonlinear correlation receivers are considered in a general context and their asymptotic performances are analyzed. It is shown that, asymptotically in the length of the spreading sequences and within some regularity on the nonlinear element, the nonlinear correlation receiver can perform with single-user error probability in a multi-user environment if the spreading sequences are asymptotically ideal. As is expected, this study involves evaluating numerous computationally cumbersome expressions. Most of the effort is thus concerned with finding efficient methods for either evaluating these expressions exactly or approximating them.

The organization of this thesis is as follows. Chapter 2 covers some preliminary results concerning the direct-sequence spread-spectrum multiple-access (DS/SSMA) signaling scheme and the model for the impulsive channel. Chapter 3 includes an overview of some results on the performance of linear and nonlinear correlation receivers in the single-user case. Chapters 4 and 5 are devoted to the exact analysis of the performance of linear and hard-limiting correlation receivers respectively, in the impulsive multi-user environment. Approximations for the error probability of these receivers are developed in Chapters 6 and 7 for systems using long spreading sequences. To gain more insight into the impulsive multi-user problem, in Chapter 8, an asymptotic analysis is carried out for both linear and nonlinear receivers, where asymptotics are taken as the spreading sequence lengths increase without bound. Finally, in Chapter 9, soft-limiting correlation receivers as an alternative for the hard-limiter are considered.

CHAPTER 2

PRELIMINARIES

In this chapter, we give a brief description of DS/SSMA signals and state some properties of this multiple-access technique. Then, a method is suggested for carrying out an impulsive channel assumption in this environment. In the process, we introduce some classes of first-order probability distribution functions that are widely used in modeling channel noise sources. Among these non-Gaussian channel models, the ε-mixture of two Gaussian distributions and the Laplacian noise models are the most interesting in the context under study here, and thus we will concentrate primarily on these models. Three alternative models, the Middleton Class A, the generalized Gaussian, and the Cauchy noise, are introduced in Appendix A.

2.1. General DS/SSMA System Model

The model of the asynchronous binary PSK direct-sequence SSMA system considered here allows K users to share a channel. The signal representing the k^{th} user's binary information sequence, $b_k(t)$, is a sequence of unit amplitude, positive and negative, rectangular pulses of duration T that can be written as

$$b_k(t) = \sum_{m=-\infty}^{\infty} b_m^{(k)} P_T(t-mT), \;\; k=1,2,\ldots,K, \tag{2.1}$$

where $b_m^{(k)} \in \{-1,+1\}$ is the m^{th} information symbol of the k^{th} user (the vector of information symbols of the k^{th} user is denoted by $\underline{b}^{(k)}$) , and $P_T(\cdot)$ is the unit rectangular pulse of duration T defined as

$$P_T(t) = \begin{cases} 1, & 0 \le t < T, \\ 0, & \text{otherwise.} \end{cases} \tag{2.2}$$

The data signal is modulated onto a phase-coded carrier, and the resulting transmitted signal for the k^{th} user is (see [26] and [27])

$$s_k(t) = \sqrt{2\pi_k}\, a_k(t)\, b_k(t) \cos(\omega_c t + \theta_k), \quad k=1,2,\ldots,K, \tag{2.3}$$

where π_k is the power utilized by the k^{th} user, ω_c is the carrier frequency, and the phase angles θ_k, $1 \le k \le K$, are not identical since the transmitters used in such systems are usually not phase synchronous. In (2.3), $a_k(t)$ is the code waveform generated by the spreading sequence assigned to the k^{th} user. This spectrum-spreading signal can be written as

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)} P_{T_c}(t - jT_c), \quad k=1,2,\ldots,K, \tag{2.4}$$

where $a_j^{(k)} \in \{-1,+1\}$ and also $a_j^{(k)} = a_{j+N}^{(k)}$ for all j and k and for some integer N. We are also assuming that N is the least period of the sequences. The quantity T_c is the chip length, and we assume that $T=NT_c$ so that there is one code period $a_0^{(k)}, a_1^{(k)}, \ldots, a_{N-1}^{(k)}$ per data symbol. For each signal, $s_k(t)$, transmitted by the k^{th} user there is an associated delay τ_k for a given receiver. This time delay accounts for the propagation delay and the lack of time synchronism between transmitters. The actual received signal for a given receiver in this model for an additive, possibly impulsive, channel is given by

$$r(t) = n(t) + \sum_{k=1}^{K} \sqrt{2P_k}\, b_k(t-\tau_k)\, a_k(t-\tau_k) \cos(\omega_c t + \phi_k), \tag{2.5}$$

where n(t) represents the channel noise, P_k is the k^{th} signal's received power, and where $\phi_k \triangleq \theta_k - \omega_c \tau_k$, $k = 1, \ldots, K$ (see Figure 2.1).

Without any loss of generality, the linear and nonlinear correlation receivers studied here are assumed to be matched to the first of the K signals in the DS/SSMA system; hence, only relative time delays and phase angles need to be considered. Therefore, we assume $\theta_1 = 0$ and $\tau_1 = 0$ in the analysis of the receiver synchronized to the first user's signal. (An interesting problem not considered in this thesis is the effect of impulsive noise on phase-locking and timing acquisition in DS/SSMA systems.) Furthermore, there is no loss of generality in assuming $\phi_k \in [0, 2\pi)$ and $\tau_k \in [0, T)$, $2 \le k \le K$, since we are concerned only with time delays modulo T and phase shifts modulo 2π.

In DS/SSMA correlation receivers the output of the correlator is sampled every T_c seconds. These samples are then passed through a memoryless nonlinearity. In these nonlinear correlation receivers, the decision on the parity of a data bit is based on the sum of N samples corresponding to that bit taken at the output of the nonlinear element (see Figure 2.2). The test statistic is thus written as

$$Y_N = \sum_{j=0}^{N-1} \hat{Z}_j^{(1)} = \sum_{j=0}^{N-1} a_j^{(1)} g(Z_j), \tag{2.6}$$

where $g(\cdot)$ is the memoryless nonlinearity (i.e., g: $R \to R$) and where Z_j and $\hat{Z}_j^{(1)}$ denote the input and output of the nonlinearity, respectively, at the j^{th} sampling instant. The additive noise in the channel is assumed to have a symmetric probability distribution and the *average* probability of error is of interest; thus, it is natural to consider only nonlinearities that are odd-symmetric around the origin. In this general setup and assuming that $\Pr(b_0^{(1)} = -1) = \Pr(b_0^{(1)} = +1) = 1/2$, the average bit-error probability using the statistic of (2.6) can be be written as

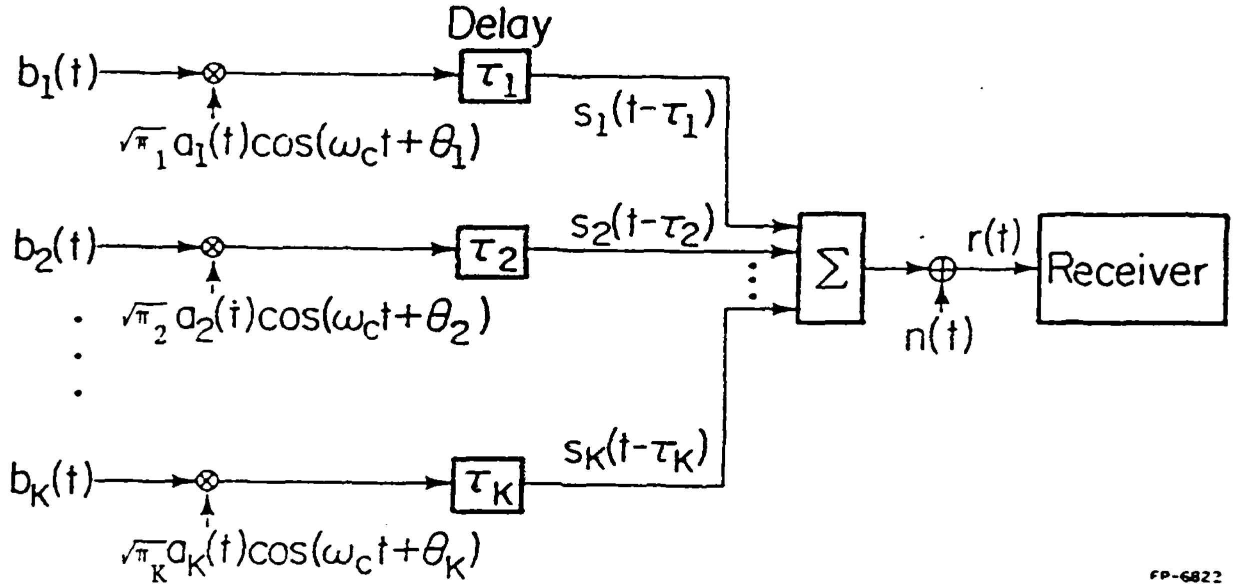

Figure 2.1. Binary direct-sequence SSMA communication system model.

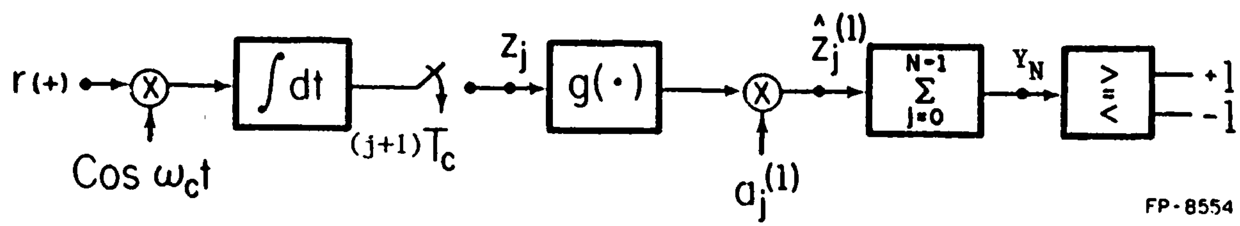

Figure 2.2. Structure of a DS/SSMA nonlinear correlation receiver.

$$\bar{P}_e = \frac{1}{2} \Pr\left[Y_N \geq 0 \mid b_0^{(1)} = -1\right] + \frac{1}{2} \Pr\left[Y_N < 0 \mid b_o^{(1)} = +1\right]. \tag{2.7}$$

We will primarily be interested in the two particular cases $g(x) = x$, corresponding to the *linear correlation receiver,* and $g(x) = \mathrm{sgn}(x)$,[1] corresponding to the *hard-limiting correlation receiver*. The reasons for choosing these receivers are the following. For AWGN channels, the linear correlation receiver provides a sufficient statistic for the single-user case, $K = 1$. Moreover, since linear correlation receivers are relatively simple to implement, the vast majority of existing direct-sequence spread-spectrum systems employ linear correlation receivers even though they may be suboptimal in a multiple-access environment. Here, we analyze the performance of this linear receiver against impulsive additive channel noise and multiple-access interference noise. The performance of the hard-limiting correlation receiver is also analyzed here since the nonlinearity is extremely easy to implement and in many communication problems it has been very effective against impulsive disturbances.

Analysis of the expression for the average error probability given in (2.7) requires some statistical assumptions on the multiple-access (MA) interference as well as a model of the channel noise. Our statistical assumptions on the MA interference are that the elements $\underline{b}^{(k)}$, τ_k, and ϕ_k, $2 \leq k \leq K$ are mutually independent random variables. We assume that $b_m^{(k)}, -\infty < m < +\infty$, is a sequence of independent data bits for each k and that $\Pr(b_m^{(k)} = +1) = \Pr(b_m^{(k)} = -1) = 1/2$ for each k and m. The random variables τ_k and ϕ_k, $2 \leq k \leq K$ are assumed to be uniformly distributed over the sets of their possible values. We also assume that these variables are independent of the channel noise and of $\underline{b}^{(1)}$.

[1] $\mathrm{sgn}(\cdot)$ denotes the signum function defined as

$$\mathrm{sgn}(x) = \begin{cases} +1 & x \geq 0 \\ -1 & x < 0 . \end{cases}$$

2.2. Impulsive Channel Models

Since communication systems are often interfered with by noises other than the classical white Gaussian noise, it is necessary to consider other appropriate (and tractable) noise models. Therefore, in this section, we introduce a tractable way of studying non-Gaussian channels for correlation receivers such as that of Figure 2.2. This method has frequently been used in various communication system analyses.

Our main assumption concerning the additive, zero mean channel noise n(t) is that samples η_j taken at the chip rate, T_c, after front-end filtering are independent and identically distributed (see (3.1) for an explicit expression for η_j). This assumption is valid when the noise process is white and Gaussian. When the noise process is white but not Gaussian, with the usual low-pass filtering the samples are uncorrelated but not necessarily independent at the appropriate sampling rate. However, making the independence assumption is still justified in our model in view of the considerable gain in tractability this assumption yields (see also [36]). This allows us to study non-Gaussian impulsive noise sources by modeling the first-order probability distribution functions of these independent random variables. Non-Gaussian modeling thus consists of considering densities that are either heavier-tailed or lighter-tailed than the Gaussian; however, our interest here is in models for impulsive non-Gaussian channels, which correspond to longer-tailed distributions.

In our analysis we have selected the first-order distribution of the random variable η_j from classes of density functions with applications in practice. For instance, communication in the low-frequency (LF) band is characterized by impulsive atmospheric noise and is distinctly non-Gaussian in nature. The fact that the receivers discussed here implement short-time correlators is advantageous since LF communication channels have inherently narrow bandwidth. Some of the models for noise sources that are considered here have been used to describe the first-order probability distribution function of impulsive noise which is common in the low-frequency band (see [8-10]). In addition to LF communications, the usual Gaussian noise assumption is inadequate in many other communication problems. Non-Gaussian impulsive models are necessary for phenomena such as atmospheric noise where lightning discharges in the vicinity of the receiver cause high amplitude noise spikes, and in underwater communication problems where the ambient acoustical noises may include impulses due to ice cracking in arctic regions. In addition to these natural non-Gaussian noise sources, there are a great variety of man-made impulsive non-Gaussian sources such as automobile ignition, other electronic devices, and heavy electrically-powered machinery.

The ε–mixture (or ε–contaminated mixture) is a commonly used and highly tractable empirical model for impulsive environments. The ε-mixture model has been frequently proposed for describing a noise environment that is nominally Gaussian with an additive impulsive noise component. The model was first applied to the detection of signals in non-Gaussian impulsive noise and has since been used in various statistical communication problems. The first-order probability density function (pdf) of this noise model has the form

$$f_{\eta_j}(x) = (1-\varepsilon)f_n(x) + \varepsilon f_I(x), \tag{2.8}$$

where $\varepsilon \in [0,1]$ and f_n and f_I are pdf's [40]. The nominal density function $f_n(.)$ is usually taken to be a Gaussian density representing background noise. The impulsive (or contaminating) component of the noise is represented by the density function $f_I(.)$ which is usually chosen to be more heavily tailed than $f_n(.)$. In (2.8), $f_I(.)$ is commonly taken to be Gaussian with large variance, and this is the model we will use here. The ratio of the variance of impulsive component to the variance of the nominal one, defined as $\gamma^2 = \sigma_I^2/\sigma_n^2$, is usually assumed to be on the order of 10 and 100. The parameter ε controls the contribution of the impulsive component to the density function. In this thesis, we will usually be interested in the effects of variations in the *shape* of a distribution on the performance of various systems rather than in the effects of simple changes in noise power. Thus, in this model we will usually vary the parameters ε and γ^2 with the total noise variance, $\sigma^2 = (1-\varepsilon)\sigma_n^2 + \varepsilon\sigma_I^2$, held constant.

Among other useful empirical models for the probability density function of η_j, the Laplacian probability density function is an example of a moderately heavy-tailed distribution. This model of impulsive noise channels belongs to a large class of distributions known as the generalized Gaussian class (see Appendix A for a complete description of the class). The Laplacian density function is given by

$$f_{\eta_j}(x) = \frac{1}{\sqrt{2}\sigma} e^{-\frac{\sqrt{2}}{\sigma}|x|} , \tag{2.9}$$

where σ^2 is the variance.

The ε-mixture and Laplacian density functions introduced above yield tractable and reasonably accurate models for most impulsive noise sources. In this thesis, we will examine the performance of multi-user communication systems against both ε-mixture and Laplacian noise channels.

CHAPTER 3

SINGLE-USER ANALYSIS IN IMPULSIVE NOISE

To gain insight into the performance of multi-user receivers in non-Gaussian impulsive noise, we first consider communications over these impulsive noise channels in the single-user context. In particular, we evaluate single-user error probabilities of linear and hard-limiting correlation receivers in ε-mixture and Laplacian channels. Appendix B contains the corresponding numerical results for the non-Gaussian channel examples introduced in Appendix A. Furthermore, in this chapter, we introduce our asymptotic analysis by considering asymptotics for the single-user problem (K=1). In the process, we state some well-known signal-detection results.

Recall from Section 2.2 that the non-Gaussian modeling applies to the samples of noise after front-end filtering. These noise samples can be expressed as (see Figure 2.2)

$$\eta'_j \triangleq \int_{jT_c}^{(j+1)T_c} n(t)\cos(\omega_c t)\,dt \ , \ \ j=0,1,\ldots,N-1 \ , \tag{3.1}$$

where the $\eta'_0, \eta'_1, \ldots, \eta'_{N-1}$ are assumed to be independent, identically distributed, random variables with zero means and variances $N_0T_c/4$. These statistical conditions would result if the input noise were white and Gaussian with spectral height $N_0/2$. Keeping these second-order conditions constant allows us to compare the results obtained here with those previously found from the study of the additive white Gaussian noise channel.

Within these assumptions, we can evaluate the performance in the single-user case of linear and hard-limiting correlation receivers when the noise samples η'_j's are distributed according to the examples in Section 2.2. Note that the spreading sequence is irrelevant to the performance in the single-user case since the noise is symmetric and the spreading sequence consists only of +1's and -1's. When K = 1 the average error probability for the conventional linear correlation receiver

(see Figure 3.1) can be computed from the formula in [12] as

$$\bar{P}_e = \frac{1}{2} - \pi^{-1} \int_0^{\infty} u^{-1} (\sin u\,)\, \Phi_2(u)\, du, \tag{3.2}$$

where $\Phi_2(u) \triangleq \left[E\{e^{iu\eta_0}\} \right]^N$ is the characteristic function of the sum of the N independent identically distributed random variables, and $\eta_0, \eta_1, \ldots \eta_{N-1}$ are given by $\eta_j = \dfrac{\eta'_j}{\sqrt{NE_b^{(1)} T_c/2}}$; $j = 0, 1, 2, \ldots N-1$ where $E_b^{(1)} \triangleq P_1 T$ is the bit energy of the user. Note that these samples of noise have been normalized to have variances $\dfrac{N_0}{2NE_b^{(1)}}$, which gives the formula (3.2).

For the example with ε-mixture channel noise, the expression (3.2) can be simplified by first noting that the characteristic function $\Phi_2(u)$ is given as

$$\Phi_2(u) = \sum_{i=0}^{N} \binom{N}{i} (1-\varepsilon)^i \varepsilon^{N-i} \exp\{-\frac{u^2 \alpha^2 \beta_i^2}{2}\}, \tag{3.3}$$

where β_i is defined as $\left[\dfrac{N[(1-\varepsilon)+\varepsilon\gamma^2]}{i+(N-1)\gamma^2} \right]^{1/2}$ and α is a function of the signal-to-noise ratio and is defined as $\alpha \triangleq \left(\dfrac{2E_b^{(1)}}{N_0} \right)^{1/2}$. Substituting (3.3) into (3.2) yields the error probability for the linear correlation receiver in ε-mixture noise as

$$\bar{P}_e = \sum_{i=0}^{N} \binom{N}{i} (1-\varepsilon)^i \varepsilon^{N-i} Q(\beta_i \alpha). \tag{3.4}$$

Similarly, for Laplacian channels, the error probability for the linear correlation receiver is computed by first writing an expression for the characteristic function as

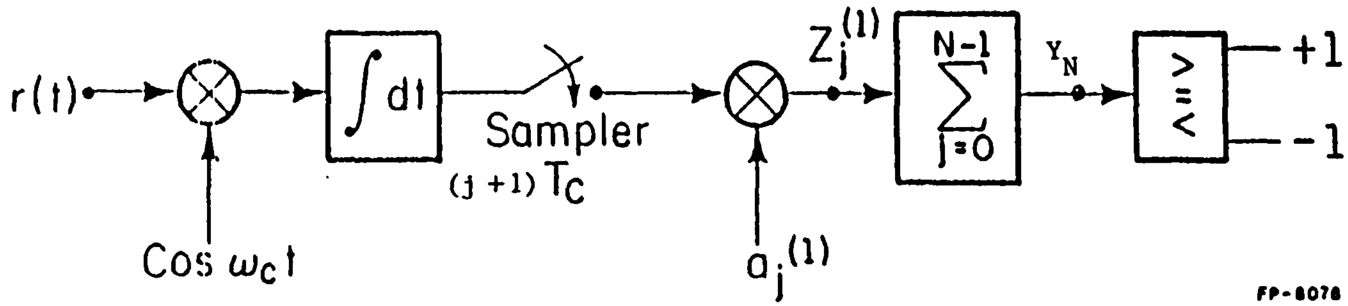

Figure 3.1. Structure of a linear correlation receiver.

$$\Phi_2(u) = \left[\frac{1}{1 + \frac{\sigma^2}{2} u^2} \right]^N \tag{3.5}$$

where σ^2 is the variance given as $\frac{N_0}{2NE_b^{(1)}}$. The single-user error probability for the Laplacian channel example is obtained by substituting (3.5) into (3.2).

To demonstrate the single-user performance of linear correlation receivers, Figures 3.2 and 3.3 have been generated for different ε-mixture channels and the Laplacian channel. The figures plot single-user average error probability versus the SNR $\triangleq 10 \log E_b^{(1)} / N_0$ (converted to dB) for the linear correlation receiver. In the ε-mixture example, the range of values for ε and γ^2 are those used in [40] and correspond to some practical examples. Comparing these impulsive non-Gaussian channel examples with the Gaussian one, these curves indicate a degradation in performance for both models over the entire range of interest of signal-to-noise ratios, with fairly large degradation in some cases. This is not surprising since the linear correlation receiver is designed to operate on the white Gaussian noise channel. These figures show that the impulsive character of the channel noise can undesirably degrade the performance of conventional linear receivers. This is in agreement with what was expected, and what has been observed in many previous studies. One interesting observation comes from comparing two curves in Figure 3.3 corresponding to examples with $\varepsilon = 0.01$, $\gamma^2 = 100$ and $\varepsilon = 0.1$, $\gamma^2 = 100$. In these examples with a fixed signal-to-noise ratio, increasing the amount of contaminated noise from $\varepsilon = 0.01$ to $\varepsilon = 0.1$ improves the performance of the linear correlator. This phenomenon occurs because the total noise variance is held constant; thus, with fixed γ^2, variation in performance is not monotonic with changes in ε. In fact two channels, one with $\varepsilon = 0.0$ and the other with $\varepsilon = 1.0$, result in identical error probabilities. Apparently a breakpoint of the error probability versus ε is near 0.01.

Typically, optimum and locally optimum detectors for non-Gaussian channels are obtained by inserting appropriate nonlinearities into the structure of the correlator as illustrated in Figure 3.2.

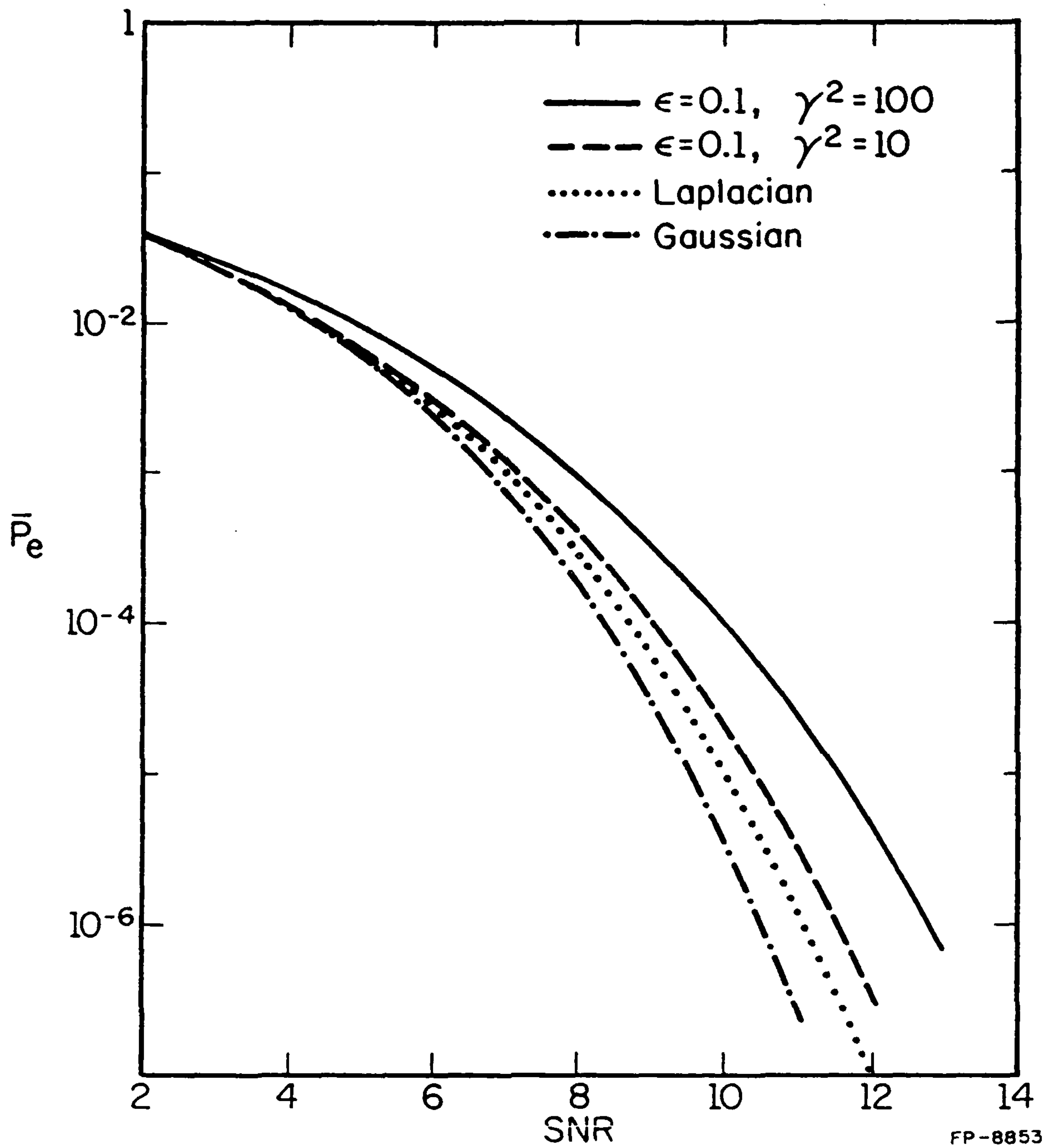

Figure 3.2. Single-user error probability for the linear correlation receiver in Gaussian, Laplacian, and ε-mixture channels, N=31.

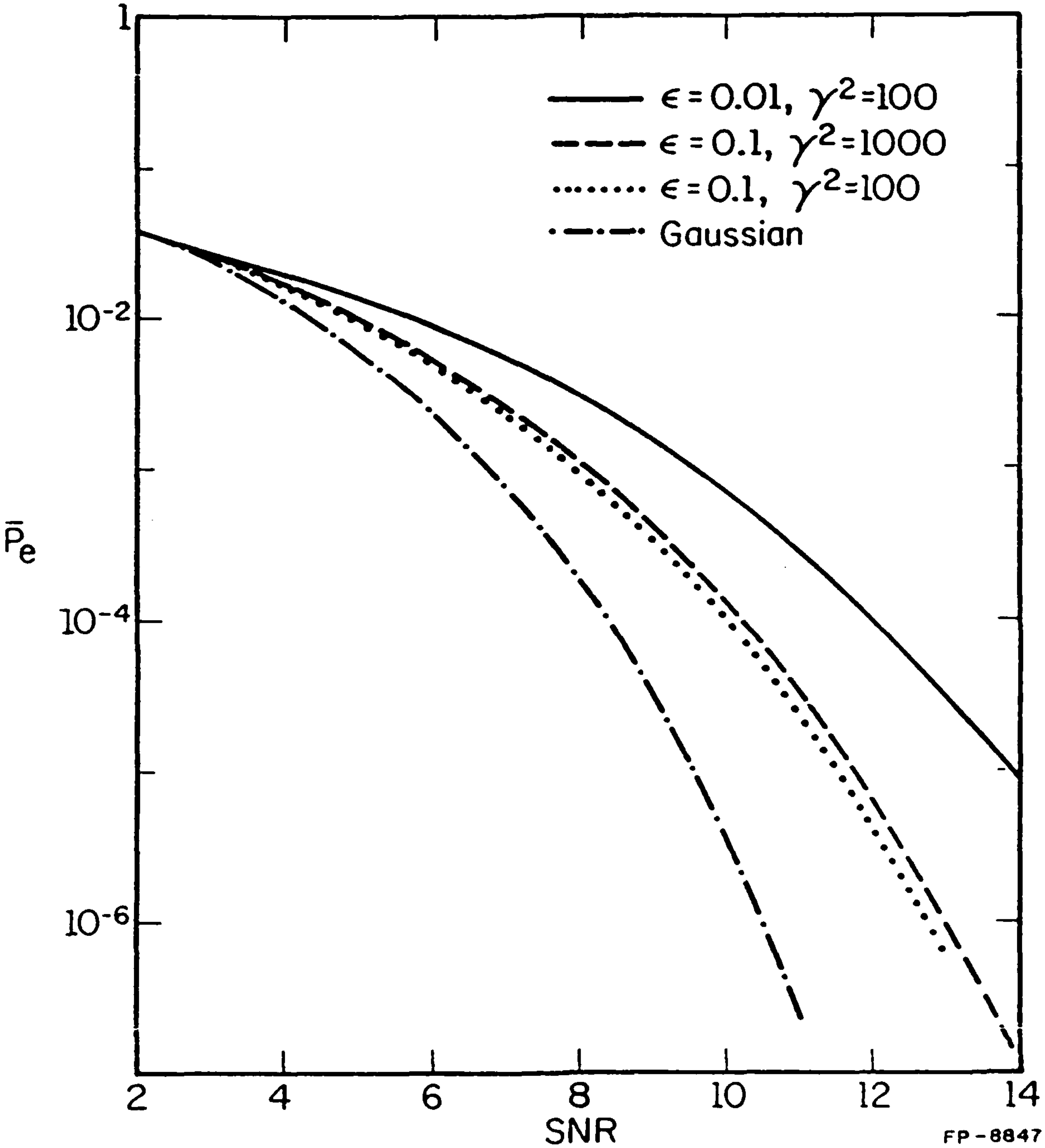

Figure 3.3. Single-user error probability for the linear correlation receiver in ε-mixture channels, N=31.

Among the many possible nonlinear correlation receivers, the hard-limiting correlator is considered here (see Figure 3.4). The hard-limiter (sign detector) is extremely easy to implement digitally and introduces virtually no processing delay since it only checks the signs of the samples. Moreover, in many communication problems, hard-limiting is known to be effective against impulsive disturbances.

In the single-user case, it is straightforward to see that $1/2\,(Y_N+N)$ formed by the test statistic of the hard-limiting receiver is binomially distributed under either bit condition. First we write the average bit error probability for the hard-limiting correlation receiver as

$$\bar{P}_e = 1/2 \sum_{m=1}^{N} \left\{ \Pr\left[Y_N=m \mid b_0^{(1)}=-1\right] + \Pr\left[Y_N=-m \mid b_0^{(1)}=+1\right] \right\}. \tag{3.6}$$

Since $1/2\,(Y_N+N)$ is binomially distributed, we have the following equality

$$\Pr\left[Y_N=m \mid b_0^{(1)}=-1\right] = \binom{N}{(N+m)/2} p^{(N+m)/2}\,(1-p)^{(N-m)/2}\ , \tag{3.7}$$

where $p \triangleq \Pr[\eta^{(1)} \geq 1]$ and where $\eta^{(1)}$ is a typical zero-mean noise sample with variance $\frac{N_0 N}{2E_b^{(1)}}$. For odd integers N, substituting (3.7) into (3.6), and noting the symmetry of the system the average bit error probability for the hard-limiting correlation receiver in the single-user case is written as

$$\bar{P}_e = \sum_{j=\frac{N+1}{2}}^{N} \binom{N}{j} p^j\,(1-p)^{N-j}. \tag{3.8}$$

Figures 3.5 and 3.6 are generated to show that hard-limiting offers substantial improvement over the linear correlation receiver for the impulsive noise examples used above over the interesting range of

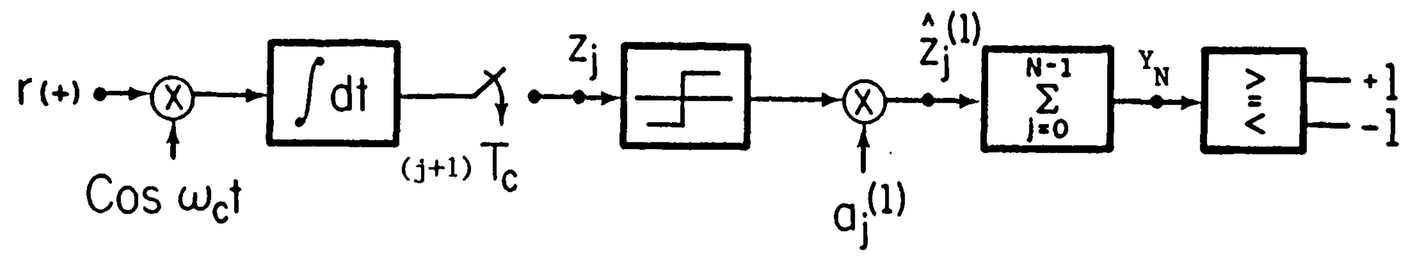

Figure 3.4. Structure of the hard-limiting correlation receiver.

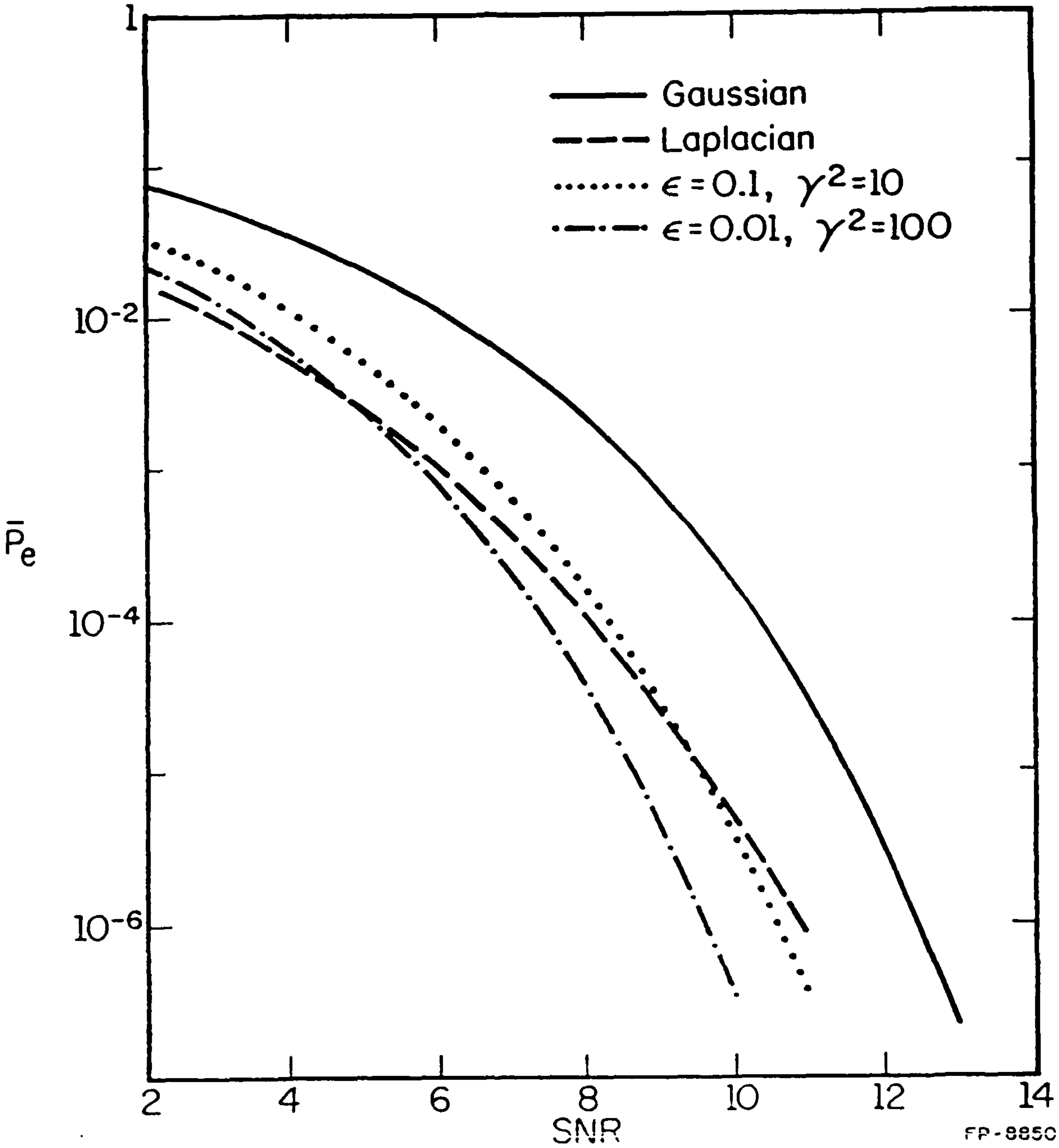

Figure 3.5. Single-user error probability for the hard-limiting correlation receiver in Gaussian, Laplacian, and ε-mixture channels, N=31.

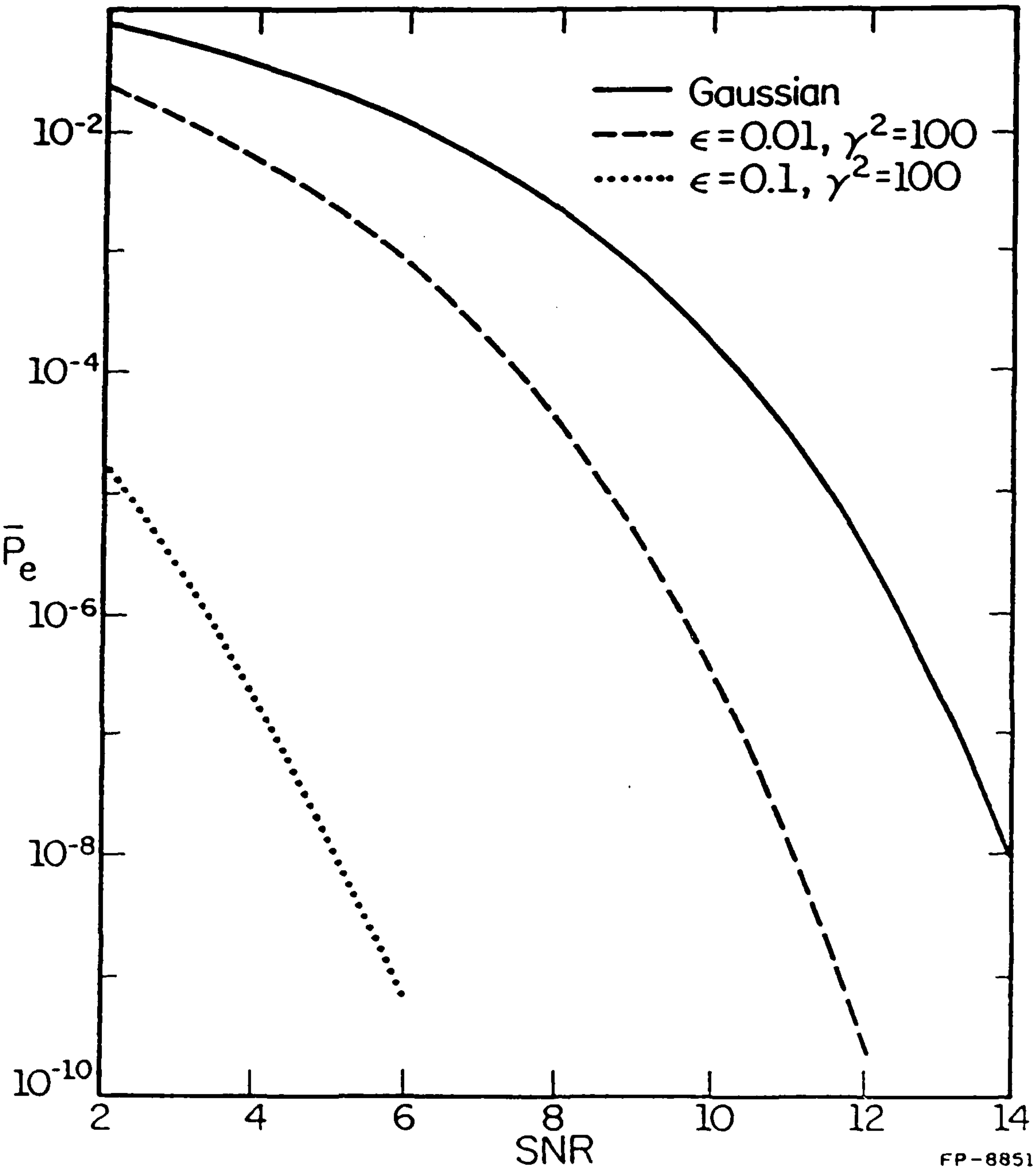

Figure 3.6. Single-user error probability for the hard-limiting correlation receiver in ε-mixture noise channels, N=31.

signal-to-noise ratios of interest. The figures plot single-user average error probability of the hard-limiting correlation receiver versus the SNR for these noise models. In Figure 3.7 the single-user error probability of the linear and hard-limiter correlation receiver is drawn versus SNR for the ε-mixture example with $\varepsilon = 0.1$ and $\gamma^2 = 100$. This is a case in which the linear correlation receiver suffered large degradation in performance compared to the Gaussian channel case. The error probabilities for the Gaussian examples are also included in the figure for comparison.

The performance of linear and hard-limiting correlation receivers in impulsive noise within the single-user context can be computed via (3.2), (3.4), and (3.8). Succeeding chapters of the thesis are devoted to analyzing the performance of these receivers in the presence of both impulsive and multi-user noises.

We now introduce an asymptotic analysis for the single-user communication problem in which we investigate the behavior of direct-sequence correlation systems when the signature-sequence length used per data bit becomes infinitely large. We keep the bit interval and the signal and noise energies *per bit* constant. In the limit, of course, these conditions would usually require infinitely large channel bandwidth, and at the receiving end, samples would be taken infinitely fast. Also note that in allowing, $N \rightarrow +\infty$, one must require the carrier frequence ω_c to increase without bound as well. Although these conditions are not realistic, they do provide information about the limiting behavior of $\bar{P}_e$ that could be useful in cases when N is very large but still finite. The multi-user situation will be treated in a later chapter.

We begin the analysis with the usual interpretation of the binary information extraction as an hypothesis testing problem. When there is only one user transmitting, deciding on the parity of a given data bit is a choice between the two hypotheses

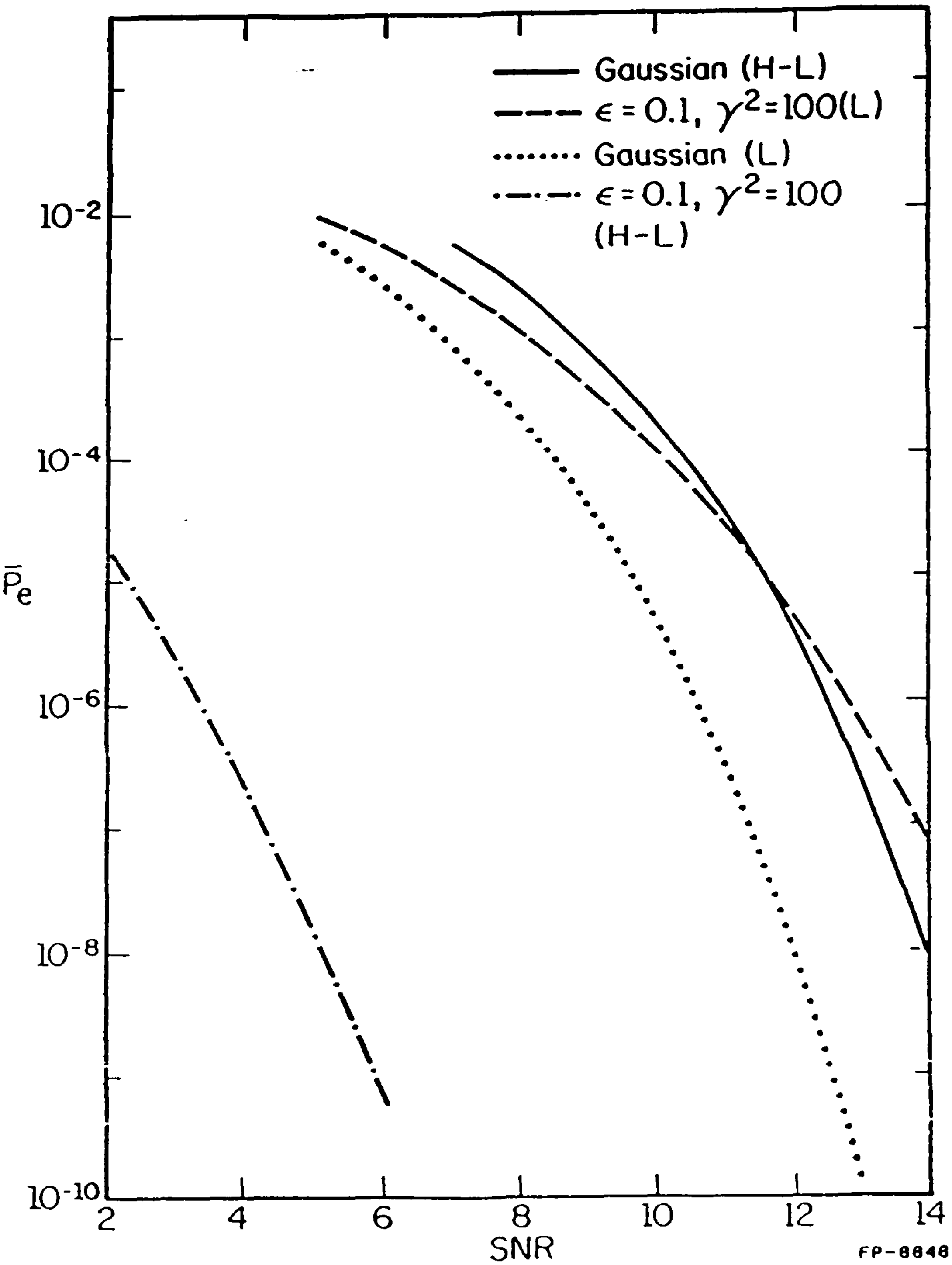

Figure 3.7. Single-user error probability for the linear (L) and hard-limiting (H-L) correlation receivers ε-mixture channels, N=31.

$$H_0 : \nu_j = \tilde{\eta}_j - \sqrt{\frac{P_1}{2N}}T$$

$$j = 0,1,2,\ldots,N-1, \tag{3.9}$$

$$H_1 : \nu_j = \tilde{\eta}_j + \sqrt{\frac{P_1}{2N}}T$$

where $\tilde{\eta}_j = \sqrt{N}\,\eta'_j$. Notice that in (3.9), $\nu_j \triangleq \sqrt{N}Z_j$ and it means the actual received signal after the front-end filter has been multiplied by $\sqrt{N}$ to expand data which otherwise were clustered around the origin. This scaling does not change the problem in any fundamental way; however, the reason for this will become clear in a moment. In (3.9) the $\tilde{\eta}_j$'s that are distributed according to examples (2.7), or (2.8) for all $0 \le j \le N-1$ have zero means and variances $\beta^2 \triangleq \frac{N_0 T}{4}$, independently of N. With this view of the problem, the test statistic based on sequence length N is $Y_N = \sum_{j=0}^{N-1} g(\nu_j)$ where, for example, g(x) equals x for the linear correlation receiver and g(x) equals sgn(x) for the hard-limiting correlation receiver. The scaling of the data by $\sqrt{N}$ allows us to analyze a fixed nonlinearity g as N changes. In other words, with T, $E_b^{(1)}$, and N_0 fixed, the common variance of the original noise samples η'_j decreases as $1/N$. In order to consider the asymptotic properties of a fixed nonlinearity, we remove this decrease by scaling η'_j to $\tilde{\eta}_j$.

Detection efficacy is a measure of asymptotic detection performance, which for the above problem is defined as (see [7], and [21-24])

$$\nu = \lim_{N \to +\infty} \left\{ \frac{\left[\dfrac{\partial E[\,Y_N \mid H_1\,]}{\partial \theta_N} \right]^2}{N\,\mathrm{Var}[\,Y_N \mid H_0\,]} \right\}, \tag{3.10}$$

where $\theta_N \overset{\Delta}{=} \sqrt{\frac{P_1}{2N}}T$. Here $E[Y_N \mid H_1]$ is the expected value of Y_N, given that the hypothesis H_1 is true and $\mathrm{Var}[Y_N \mid H_0]$ is the variance of Y_N, given that the hypothesis H_0 is true. One significance of detection efficacy is that, within mild regularities on the nonlinearity g and the noise distribution, it relates to the asymptotic error probability via

$$\lim_{N\to+\infty} \overline{P}_e = Q\left\{T\sqrt{\frac{P_1}{2}\nu}\right\}. \tag{3.11}$$

For the particular nonlinearities of interest here, (3.11) is valid for all of the impulsive noise models under consideration. The efficacies are $\nu = \frac{4}{N_0 T}$ for the linear correlator and $\nu = 4f_\eta^2(0)$ for the hard-limiting correlation receiver, where $f_\eta(.)$ is the probability density function of the scaled sampled noise.

Recall that in this asymptotic analysis the energy per data bit for the user $E_b^{(1)}$, the length of the data period T, the noise power, and hence the signal-to-noise ratio are kept constant. Table 3.1 contains the asymptotic average bit-error probabilities for the linear correlator and the hard-limiting correlation receiver in the single-user case with an SNR of 8 dB. Figures 3.8 and 3.9 are drawn to exhibit the asymptotic performance of these receivers over the interesting range of signal-to-noise ratios in non-Gaussian impulsive noise. These results indicate considerable improvement in the asymptotic performance by using hard-limiting correlation receivers in place of linear correlators for impulsive noise channels for the entire range of interest of signal-to-noise ratios. Note that this improvement is generally more pronounced than in the N = 31 case treated above, particularly for the Laplacian case.

TABLE 3.1. ASYMPTOTIC ERROR PROBABILITY FOR LINEAR AND HARD-LIMITING CORRELATION RECEIVERS IN GAUSSIAN, LAPLACIAN, AND ε-MIXTURE CHANNELS, SNR=8 dB.

DISTRIBUTIONS	LINEAR	HARD-LIMITER
GAUSSIAN	1.91×10^{-4}	2.30×10^{-3}
LAPLACIAN	1.91×10^{-4}	2.53×10^{-7}
ε-MIXTURE		
$\varepsilon = 0.1$ $\gamma^2 = 10$	1.91×10^{-4}	1.36×10^{-4}
$\varepsilon = 0.01$ $\gamma^2 = 100$	1.91×10^{-4}	3.71×10^{-5}
$\varepsilon = 0.1$ $\gamma^2 = 100$	1.91×10^{-4}	8.29×10^{-18}

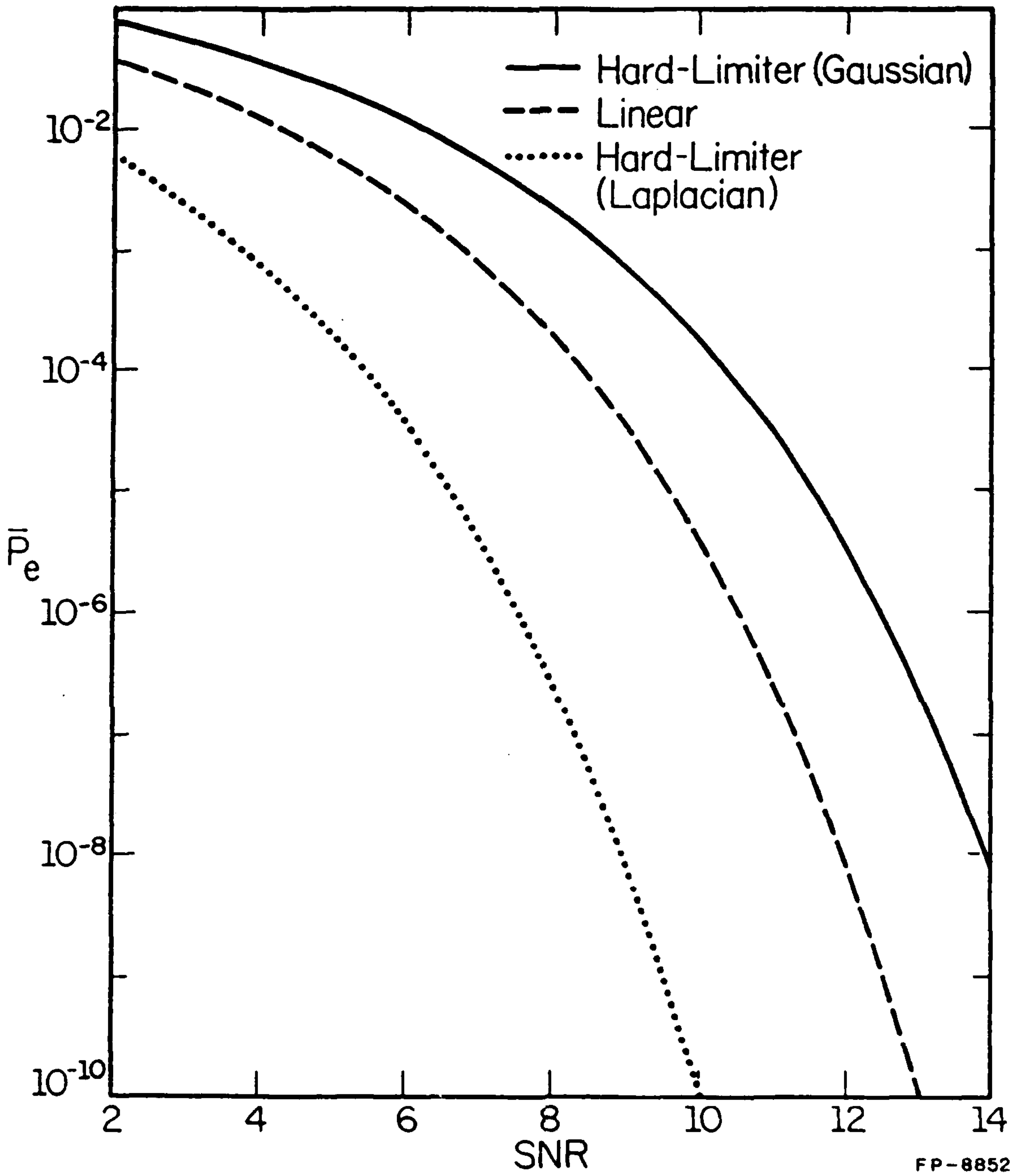

Figure 3.8. Asymptotic error probability for the linear correlation receiver, and hard-limiting correlation receiver in Gaussian and Laplacian channels.

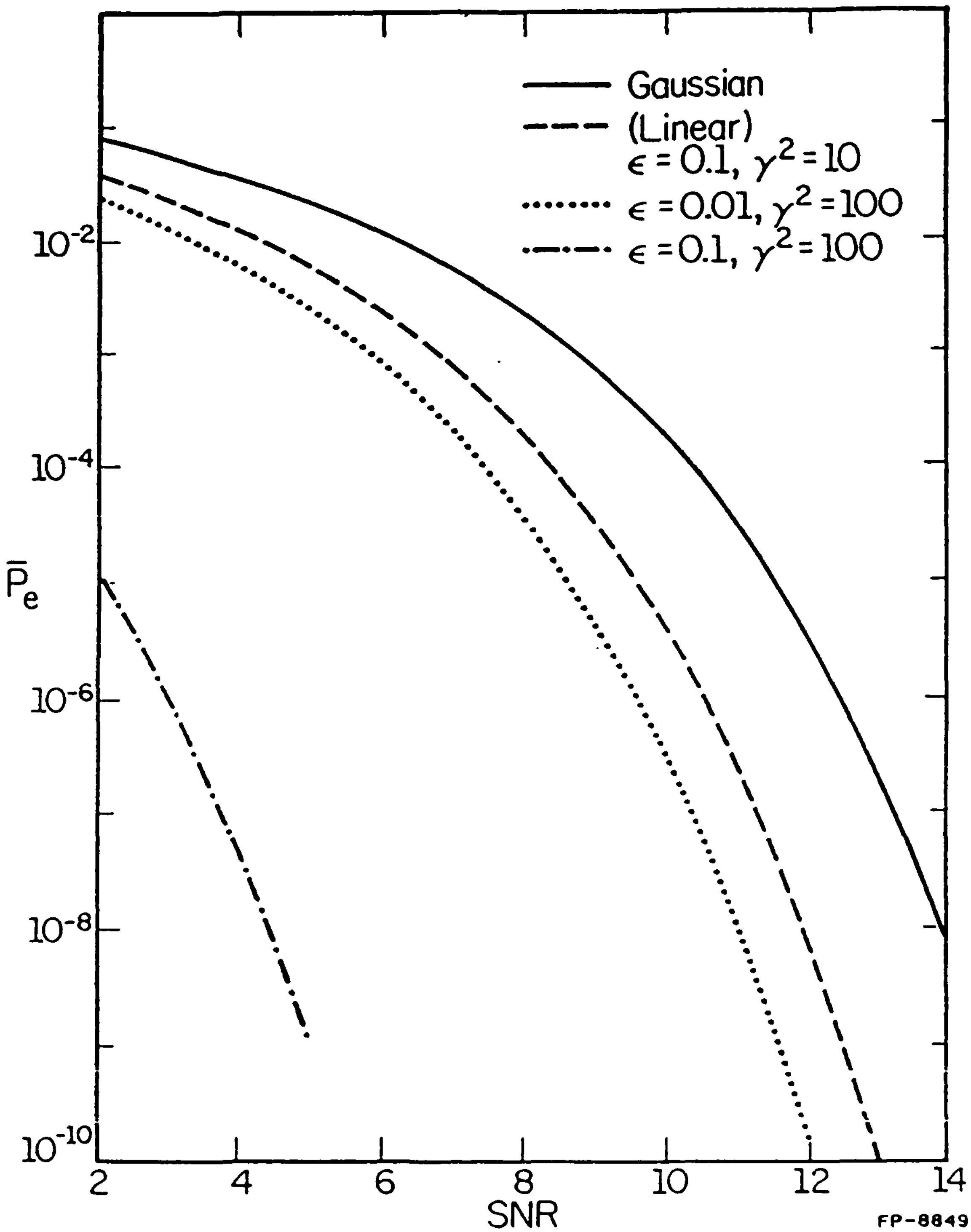

Figure 3.9. Asymptotic error probability for the hard-limiting correlation receiver in ε-mixture noise channels.

CHAPTER 4

ANALYSIS OF LINEAR CORRELATION RECEIVERS IN IMPULSIVE AND MULTI-USER NOISES

In this chapter, we study the performance of linear receivers in the direct-sequence multi-user environment over impulsive-noise channels. Appendix B contains the corresponding results for the non-Gaussian channel examples introduced in Appendix A. Here, in particular, we wish to study linear digital correlation receivers (see Figure 3.1) matched to the first of K users. In such receivers, the output of the chip-matched filter is sampled every T_c seconds. The test statistic is the sum of N of these samples,

$$Y_N = \sum_{j=0}^{N-1} Z_j^{(1)}, \tag{4.1}$$

where $Z_j^{(1)}$ is $a_j^{(1)}$ times the sampled output of the front-end filter. When the input signal is the noisy multi- user signal of (2.5), $Z_j^{(1)}$ is given by

$$Z_j^{(1)} = \eta'^{(1)}_j + \sum_{k=2}^{K} \sqrt{\frac{P_k}{2}} \cos\phi_k \int_{jT_c}^{(j+1)T_c} a_j^{(1)}\, a_k(t-\tau_k)\, b_k(t-\tau_k)\, dt + \sqrt{\frac{P_1}{2}}\, T_c\, b_0^{(1)} ,$$

$$j=0,1,\ldots,N-1, \tag{4.2}$$

where double-frequency terms have been ignored, and $\eta'^{(1)}_j$, is defined as

$$\eta'^{(1)}_j \triangleq \int_{jT_c}^{(j+1)T_c} n(t)\, a_j^{(1)} \cos \omega_c t\, dt, \quad j=0,1,\ldots,N-1. \tag{4.3}$$

The noise components of the sampled signals, $\eta'^{(1)}_0, \ldots, \eta'^{(1)}_{N-1}$, are assumed to be independent and identically distributed random variables with zero means and variances $\frac{N_0 T_c}{4}$. It is easy to see that Y_N given in (4.1) is not a sufficient statistic for deciding all symbols (for $K > 1$) even for the Gaussian case. For this problem, a more complex receiver involving sequence detection may provide a statistic which permits a more nearly optimal decision (see, e.g., [41-42]). However, correlation receivers perform acceptably and are very simple to implement, and thus they have been used in most direct-sequence SSMA systems, even though they are suboptimal in a multiple-access environment.

To recognize the contribution of the non-Gaussian noise and the interference from other users in the bit error probability, and since for the linear correlator we can write $Y_N = \int_0^T r(t)\, a_1(t) \cos\omega_c t\, dt$, the average error probability of the linear correlation receiver is given by (see [12] for details)

$$\overline{P}_e = \frac{1}{2} - \frac{1}{2} \Pr\left[-1 \le \eta_s + I_1 < +1\right], \tag{4.4}$$

where $\eta_s \triangleq \sum_{j=0}^{N-1} \frac{\eta'^{(1)}_j}{T\sqrt{\frac{P_1}{2}}}$, and where the normalized interference term I_1 is given by

$$I_1(\underline{b}, \underline{\tau}, \underline{\phi}) = \sum_{k=2}^{K} \sqrt{\epsilon_{k,1}}\, \frac{\cos\phi_k}{T} \left\{ b_{-1}^{(k)} R_{k,1}(\tau_k) + b_0^{(k)} \hat{R}_{k,1}(\tau_k) \right\}, \tag{4.5}$$

where $\epsilon_{k,1}$, is defined as the ratio of the powers $\frac{P_k}{P_1}$, for $k = 2, 3, \ldots, K$. The two terms $R_{k,1}$ and $\hat{R}_{k,1}$ appearing in (4.5) are the continuous-time partial cross-correlation functions between the k^{th} and first code waveforms, defined by (see also [27])

$$R_{k,1}(\tau) = \int_0^{\tau} a_k(t-\tau)a_1(t)dt\,, \tag{4.6}$$

and

$$\hat{R}_{k,1}(\tau) = \int_{\tau}^{T} a_k(t-\tau)a_1(t)dt\,, \tag{4.7}$$

for $0 \le \tau \le T$. For values of τ in the range $0 \le mT_c \le \tau \le (m+1)T_c \le T$, the two cross-correlation functions $R_{k,1}$, and $\hat{R}_{k,1}$ can be written as

$$R_{k,1}(\tau) = C_{k,1}(m-N)T_c + [C_{k,1}(m+1-N) - C_{k,1}(m-N)](\tau - mT_c)\,, \tag{4.8}$$

and

$$\hat{R}_{k,1}(\tau) = C_{k,1}(m)T_c + [C_{k,1}(m+1) - C_{k,1}(m)](\tau - mT_c)\,, \tag{4.9}$$

where $C_{k,1}$ is the discrete aperiodic cross-correlation function for the sequences $\underline{a}^{(k)}$ and $\underline{a}^{(1)}$ defined by

$$C_{k,1}(m) = \begin{cases} \sum\limits_{j=0}^{N-1-m} a_j^{(k)}\, a_{j+m}^{(1)} & 0 \le m \le N-1 \\ \sum\limits_{j=0}^{N-1+m} a_{j-m}^{(k)}\, a_j^{(1)} & 1-N \le m < 0\;. \\ 0 & |m| \ge N \end{cases} \tag{4.10}$$

Recalling the statistical assumptions on the asynchronous DS/SSMA system, we note that I_1 has a symmetric distribution, and since η_s and I_1 are assumed to be independent, the average error probability can be written as

$$\bar{P}_e = \frac{1}{2} - \pi^{-1}\int_0^{\infty} u^{-1} (\sin u)\, \Phi_2(u)\, du + \pi^{-1}\int_0^{\infty} u^{-1} (\sin u)\, \Phi_2(u)\, [1-\Phi_1(u)]\, du, \tag{4.11}$$

where $\Phi_2(u) \triangleq E\{e^{iu\eta_s}\} = [E\{e^{iu\eta_0}\}]^N$ as in (3.2) and $\Phi_1(u) \triangleq E\{e^{iuI_1}\}$. Since $\bar{P}_e$ is written conveniently in terms of characteristic functions, contributions of the multiple-access (MA) interference and noise are distinguishable. Note that the first two terms of the expression for the multi-user error probability are identical to the right-hand side of (3.2). The characteristic functions $\Phi_2(u)$ are given in the previous chapter for all noise models under consideration.

With regard to $\Phi_1(u)$ in (4.11), since the K−1 multiple-access interference terms

$$I_{k,1}(b_k, \tau_k, \phi_k) \triangleq \sqrt{\epsilon_{k,1}}\, \frac{\cos\phi_k}{T} \left\{ b_{-1}^{(k)} R_{k,1}(\tau_k) + b_0^{(k)} \hat{R}_{k,1}(\tau_k) \right\}, \; 2 \le k \le K \tag{4.12}$$

(i.e., these form the total multiple-access interference as $I_1 = \sum_{k=2}^{K} I_{k,1}(b_k, \tau_k, \phi_k)$) are mutually independent, we can write (see also [12])

$$\Phi_1(u) = \prod_{k=2}^{K} \left\{ (8\pi T)^{-1} \sum_{b_{-1}, b_0} \int_0^{2\pi} \int_0^{T} \exp[\, iuI_{k,1}(b_k, \tau_k, \phi_k)\,] d\tau d\phi \right\}, \tag{4.13}$$

where $\sum_{b_{-1},b_0}$ denotes the sum over all $(b_{-1}^{(k)}, b_0^{(k)})$ with $b_l^{(k)} \in \{+1, -1\}$. The expression in (4.13) can be written as

$$\Phi_1(u) = \prod_{k=2}^{K} \frac{1}{2N} \left\{ \sum_{m=0}^{N-1} [f(u,m,\sqrt{\epsilon_{k,1}}\,\theta_{k,1}) + f(u,m,\sqrt{\epsilon_{k,1}}\,\hat{\theta}_{k,1})] \right\}, \tag{4.14}$$

where

$$f(u,m,y) \triangleq \frac{1}{2\pi T_c} \int_0^{2\pi} \int_0^{T_c} \exp\left\{ iu\, [y(m+1)\tau + y(m)(T_c-\tau)]\, \frac{\cos\theta}{T} \right\} d\tau\, d\theta\,, \tag{4.15}$$

for an arbitrary function y. In (4.14) $\theta_{k,1}$ and $\hat{\theta}_{k,1}$ are the periodic and odd cross-correlation functions for the binary spreading sequences, respectively, and they are defined as (see [27] and [33])

$$\theta_{k,1}(m) \triangleq C_{k,1}(m) + C_{k,1}(m-N)\,, \tag{4.16}$$

and

$$\hat{\theta}_{k,1}(m) \triangleq C_{k,1}(m) - C_{k,1}(m-N)\,, \tag{4.17}$$

for $0 \le m \le N-1$. For the binary PSK system considered here, (4.15) reduces to the following expression which was obtained in [12] and [11] :

$$f(u,m,y) \triangleq \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos[\frac{u}{2N}(y(m+1)+y(m))\cos\theta]\, \mathrm{sinc}[\frac{u}{2\pi N}(y(m+1)-y(m))\cos\theta\,]\, d\theta \quad . \tag{4.18}$$

Typical average bit error probabilities of the linear correlation receiver for the two-user ($K = 2$) case with various values of distribution parameters obtained by substituting (4.14) and the corresponding expression for Φ_2 in (4.11) are contained in Table 4.1. For an example with $N = 31$ a

TABLE 4.1. AVERAGE ERROR PROBABILITY OF LINEAR CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN, LAPLACIAN, AND ε-MIXTURE NOISE CHANNELS, SNR=8.0 dB.

DISTRIBUTION	K	N=31	N=63
GAUSSIAN	1	1.91×10^{-4}	1.91×10^{-4}
	2	4.16×10^{-4}	3.06×10^{-4}
LAPLACIAN	1	2.49×10^{-4}	
	2	5.45×10^{-4}	
ε-MIXTURE 0.1,10	1	4.02×10^{-4}	2.93×10^{-4}
	2	6.76×10^{-4}	4.21×10^{-4}
0.01,100	1	3.02×10^{-3}	1.55×10^{-3}
	2	3.46×10^{-3}	1.76×10^{-3}
0.1,100	1	9.47×10^{-4}	5.61×10^{-4}
	2	1.31×10^{-3}	7.19×10^{-4}

spreading sequence (also known as signature sequence) of length 31 is assigned to each user corresponding to top entries of Table A.1(a) of [28]. These are auto-optimal least-sidelobe energy (AO/LSE) phases of maximal-length sequences (m-sequence) of period N. The results presented here are typical of our findings for a variety of system parameters. For simplicity, the examples are carried through under the assumption that all users have the same power. All of these results are obtained with fixed signal-to-noise ratio, thereby showing the effects of the shape of the noise density on performance. For the more interesting example, the ε-mixture case, Figure 4.1 is generated to illustrate the performance for a wide range of values for SNR. Also shown on Figure 4.1 is the performance obtained in a six-user case Gaussian channel.

The principal conclusion here is that, as expected, the linear receiver does not perform as well as the Gaussian model predicts when the non-Gaussian noise has an impulsive nature (heavy-tailed distribution). Also, the effects of the impulsive noise on the two-user system seems to be nearly the same as that on the single-user system. Similar to the single-user case, the multi-user performance of linear correlation receiver in the ε-mixture channel with $\varepsilon = 0.1$, $\gamma^2 = 100$ is better than that in the channel with $\varepsilon = 0.01$, $\gamma^2 = 100$. As a final remark on these results, in Figure 4.1 we see that for most values of SNR the error probability for the linear correlation receiver in one of the ε-mixture examples with two users is nearly as high as that for the Gaussian example with six users. This indicates that if the Gaussian assumption for a channel is violated in favor of *this* ε-mixture, the resulting degradation in performance is equivalent to that caused by subscribing four additional users to a Gaussian channel.

Next we consider the near-far effects on the performance of the linear DS/SSMA correlation receiver in an impulsive channel. We assume that two users share an ε-mixture channel and that the interfering user has power different than the receiver one (i.e., $\epsilon_{2,1} = \frac{P_2}{P_1} \neq 1$). Results for a fixed signal-to-noise ratio (of user 1) equal to 8 dB and a value of N=31 are contained in Table 4.2 and Figure 4.2. The error probabilities indicate smaller *degradation* in performance due to the impulsive

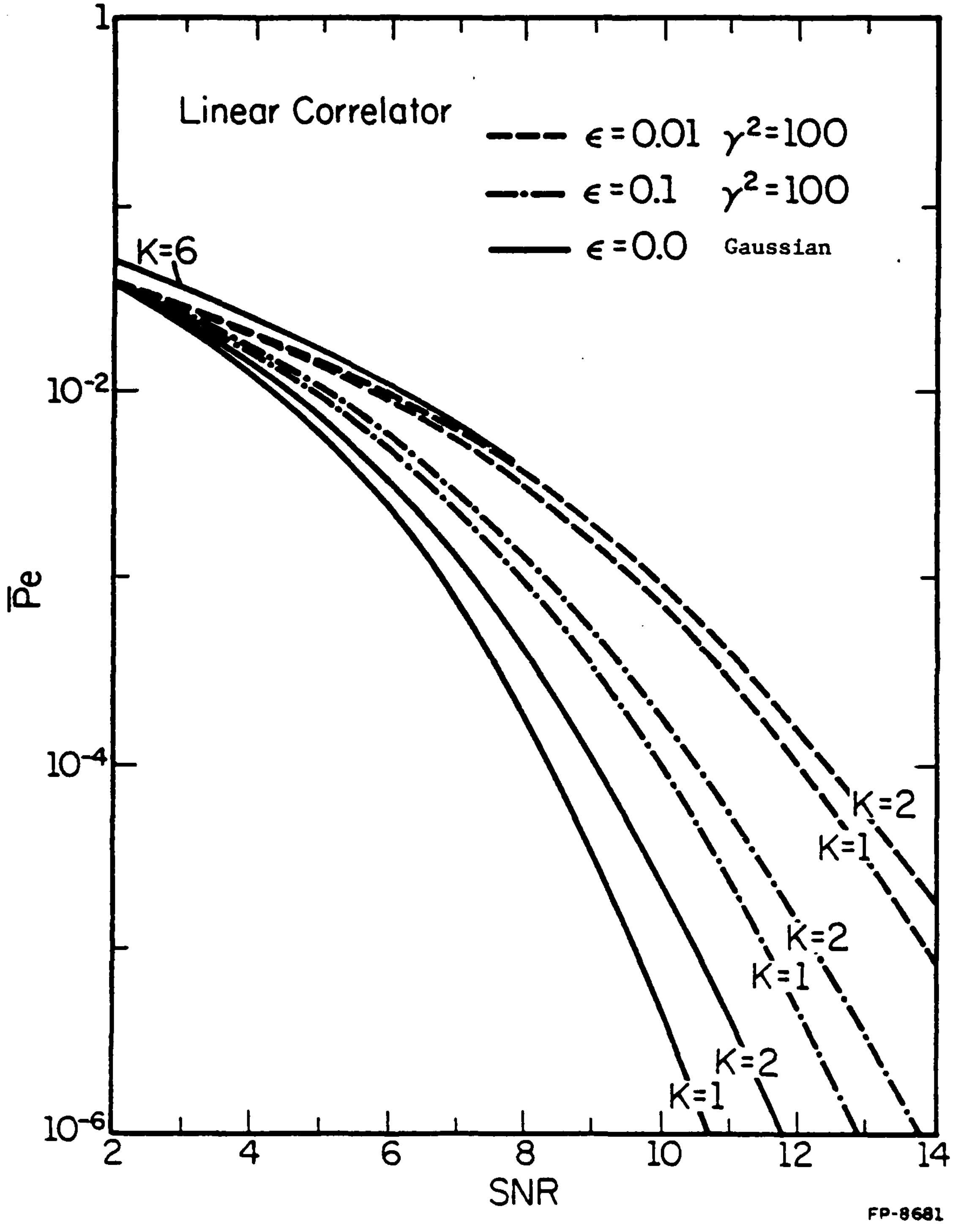

Figure 4.1. Error probability for the linear DS/SSMA correlation receiver in ε-mixture channels, N=31.

TABLE 4.2. ERROR PROBABILITIES OF LINEAR CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND ε-MIXTURE CHANNELS, SNR=8.0 dB, N=31, K=2, TWO USERS WITH UNEQUAL POWERS ($\epsilon_{2,1} = P_2/P_1 \geq 1$).

DISTRIBUTION	$\epsilon_{2,1} = 0.0dB$	$\epsilon_{2,1} = 3.01dB$	$\epsilon_{2,1} = 4.77dB$
GAUSSIAN	4.16×10^{-4}	7.84×10^{-4}	1.32×10^{-3}
ε-mixture			
ε=0.1, γ^2=10	6.76×10^{-4}	1.08×10^{-3}	1.63×10^{-3}
ε=0.01, γ^2=100	3.46×10^{-3}	3.94×10^{-3}	4.49×10^{-3}
ε=0.1, γ^2=100	1.31×10^{-3}	1.79×10^{-3}	2.39×10^{-3}

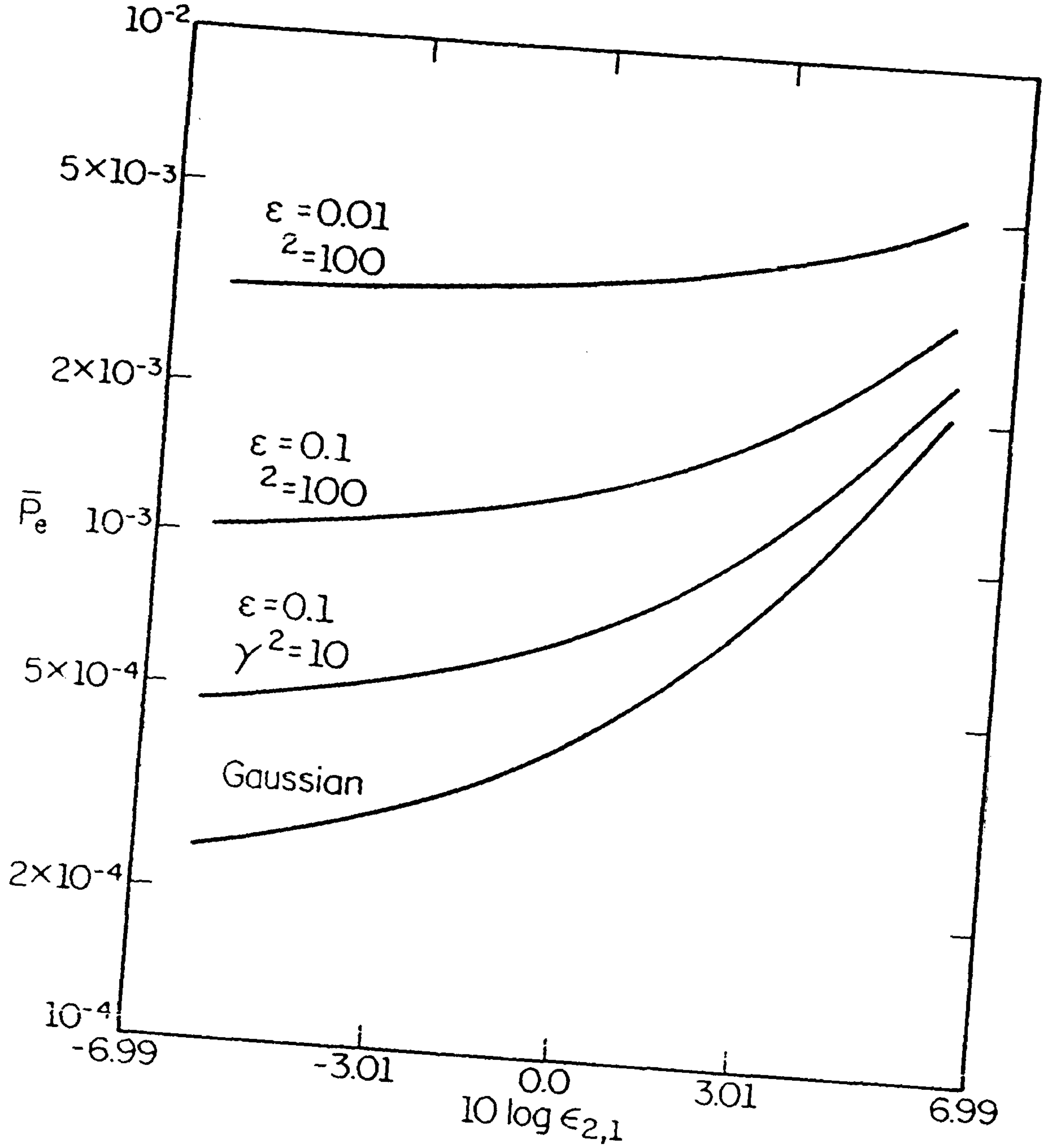

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Figure 4.2. Near-far effects in the error probability of the linear correlation receiver in ε-mixture noise channels, SNR (of user 1) =8 dB, K=2, N=31.

noise component as the other user's power increases when compared to the equal power example. This is due to the fact that, as the other user's interference becomes more significant, the channel noise represents a smaller fraction of the total noise. Consequently, the performance of the linear correlation receiver in these impulsive channels becomes comparable to the performance of that receiver in a Gaussian channel under similar conditions.

Summarizing this chapter, we have considered the performance of linear correlation reception in a typical two-user DS/SSMA system operating in impulsive noise. As expected, we have seen that performance in this situation can degrade undesirably from that predicted for the Gaussian noise, particularly when the channel noise is not dominated by the multi-user interference.

Aside from the these findings it should be noted that we experienced a rapid growth in the computational effort required to compute $\overline{P}_e$ for larger N, as one would expect. The problem of approximating $\overline{P}_e$ for large values of N will be discussed in Chapter 6.

CHAPTER 5

ANALYSIS OF HARD-LIMITING CORRELATION RECEIVERS IN IMPULSIVE MULTI-USER CHANNELS

Motivated by the finding of the previous chapters, in this chapter we study the performance of the hard-limiting correlation receiver under the same conditions as those for which the linear correlation receiver was analyzed. We introduce limiting to diminish the influence of large samples (e.g., noise impulses) on the test statistic. In Chapter 3 we showed that, in the single-user environment, hard-limiting correlation receivers can be used to suppress the effects of non-Gaussian impulsive noise and thus to achieve a lower average probability of error. Here, we analyze the performance of the hard-limiting correlation receiver in the presence of non-Gaussian impulsive noise and multi-user interference, and we compare these combined effects with those on the linear correlation receiver. As noted before, the nonlinear element in the structure of the hard-limiting receiver is extremely easy to implement digitally and virtually no processing delay is introduced since it only checks the signs of the samples, Z_j's (see Figure 3.4).

The average bit-error probability for the hard-limiting receiver can be derived from the general form (2.7). We assume throughout that the number of chips per bit is odd (this is usually the case as N is usually 2^n-1 for some integer n) and so since $\mathrm{sgn}(x) \in \{-1,+1\}$, the test statistic is an odd integer, $Y_N \in \{-N,-N+2,\ldots,-3,-1,+1,3,\ldots,N\}$.

It is straightforward to see that the test statistic $1/2(Y_N+N)$ formed by the hard-limiting receiver is multinomially distributed under either bit condition. From this it follows that the average Sor probability for the hard-limiting correlation receiver can be written as

$$\bar{P}_e = E_{\underline{\tau},\underline{\phi},\underline{b}} \left\{ \tfrac{1}{2} \sum_{m=1}^{(N+1)/2} \left[\Pr\left[Y_N = 2m-1 \mid b_0^{(1)} = -1, \underline{\tau}, \underline{\phi}, \underline{b} \right] \right. \right. \tag{5.1}$$

$$\left. \left. + \Pr\left[Y_N = -2m+1 \mid b_0^{(1)} = +1, \underline{\tau}, \underline{\phi}, \underline{b} \right] \right] \right\},$$

where $\underline{\tau} = (\tau_2, \ldots, \tau_K)$ with $\tau_k \in [0,T)$, $k = 2, \ldots, K$, $\underline{\phi} = (\phi_2, \ldots, \phi_K)$ with $\phi_k \in [0, 2\pi)$, $k = 2, \ldots, K$, and $\underline{b} = [(b_{-1}^{(2)}, b_0^{(2)}), \ldots, (b_{-1}^{(K)}, b_0^{(K)})]$ with $b_l^{(k)} \in \{-1, +1\}$, $k = 2, \ldots, K$.

We now consider two methods for computing the average Sor probability (5.1) for the hard-limiting correlation receiver in multiple-access noise. Results computed from these will then be compared with analogous results for the linear correlation receiver from Chapter 4.

5.1. The Characteristic Function Method

Our first method for computation of $\bar{P}_e$ for the hard-limiting correlator uses properties of the characteristic functions of discrete random variables. The characteristic function of Y_N conditioned on $(\underline{\tau}, \underline{\phi}, \underline{b})$ and $b_0^{(1)}$ is given by

$$\Phi_{Y_N}\Big|_{b_0^{(1)}, \rho_I}(u) = \prod_{j=0}^{N-1} \left[e^{iu} p_j^{b_0^{(1)}} + e^{-iu}(1 - p_j^{b_0^{(1)}}) \right], \tag{5.2}$$

where $p_j^{b_0^{(1)}} \triangleq \Pr\left[Z_j^{(1)} \geq 0 \mid b_0^{(1)}, \rho_I \right]$, and where ρ_I is used to denote the collection of variables $(\underline{\tau}, \underline{\phi}, \underline{b})$. Note that the probability of a discrete random variable taking an integer value m is given in terms of its characteristic function as

$$\Pr(\,Y_N = m\,) = \frac{1}{2\pi}\int_0^{2\pi} e^{-ium}\,\Phi_{Y_N}(u)\,du; \tag{5.3}$$

i.e., it is the m^{th} Fourier coefficient of the characteristic function. This expression can be simplified using some properties of the characteristic function Φ_{Y_N}. First note that $\Phi_{Y_N}(u) = -\Phi_{Y_N}(u+\pi)$ and $\Phi_{Y_N}(u) = -\bar{\Phi}_{Y_N}(\pi-u)$, because N is assumed to be an odd integer. Now since in (5.1) the summation is taken over all the odd integers m between 1 and N, we only need to consider odd integers m. Equation (5.3) can now be simplified as

$$\Pr(Y_N = m) = \frac{1}{\pi}\left[\int_0^{\frac{\pi}{2}} e^{-ium}\,\Phi_{Y_N}(u)\,du + \int_0^{\frac{\pi}{2}} e^{ium}\,\bar{\Phi}_{Y_N}(u)\,du\right]. \tag{5.4}$$

Moreover, for all odd integers m it follows that

$$\Pr\left[Y_N = m \mid b_0^{(1)} = -1, \rho_I\right] = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{Re}\left\{e^{-ium}\,\Phi_{Y_N}(u \mid -1, \rho_I)\right\}du\,. \tag{5.5}$$

Before substituting (5.2) into (5.5), we note the following :

$$e^{-ium}\,\Phi_{Y_N}(u) = \prod_{j=0}^{N-1}\left\{[\,\cos u(1+\frac{m}{N}) + 2p_j \sin u \sin(\frac{um}{N})\,]\right. \tag{5.6}$$

$$\left. -\,i\,[\,\sin u(1+\frac{m}{N}) - 2p_j \sin u \cos(\frac{um}{N})\,]\right\},$$

where $p_j \overset{\Delta}{=} p_j^{b_0^{(1)}}$ with $b_0^{(1)} = -1$. Substituting (5.6) into (5.5) and after a few straightforward intermediate steps, the expression (5.3) can be written as

$$\Pr\left[Y_N = m \mid b_0^{(1)} = -1, \rho_I\right] = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \prod_{j=0}^{N-1} R(u, p_j) \cos\left[\sum_{j=0}^{N-1} \Delta(u, p_j, m)\right] du\,, \tag{5.7}$$

where

$$R(u,p) \triangleq [1 + 4p(p-1)\sin^2 u]^{\frac{1}{2}}\,, \tag{5.8}$$

and

$$\Delta(u,p,m) \triangleq \tan^{-1}\left\{\frac{\sin u(1+\frac{m}{N}) - 2p \sin u \cos \frac{um}{N}}{\cos u(1+\frac{m}{N}) + 2p \sin u \sin \frac{um}{N}}\right\}. \tag{5.9}$$

Similarly, we can write

$$\Pr\left[Y_N = -m \mid b_0^{(1)} = +1, \rho_I\right] = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \prod_{j=0}^{N-1} R(u, \hat{p}_j) \cos\left[\sum_{j=0}^{N-1} \Delta(u, \hat{p}_j, m)\right] du\,, \tag{5.10}$$

where $\hat{p}_j \triangleq \Pr[\, Z_j^{(1)} < 0 \mid b_0^{(1)} = +1, \rho_I\,]$.

At this point, for simplicity in the analysis, we assume that relative time delays $\underline{\tau}$ among the users are fixed. This analysis can be applied to a sufficiently rich set of delays to obtain an approximation to the average bit-error probability. We write the probabilities p_j and $\hat{p}_j$ explicitly as functions of $\underline{\tau}$, via

$$p_j(\underline{\tau}) \overset{\Delta}{=} \Pr\left[\eta_j^{(1)} + I_j^{(1)}(\underline{b}, \underline{\tau}, \underline{\phi}) \geq 1 \mid \underline{\tau}, \underline{b}, \underline{\phi}\right]$$

$$\hat{p}_j(\underline{\tau}) \overset{\Delta}{=} \Pr\left[\eta_j^{(1)} + I_j^{(1)}(\underline{b}, \underline{\tau}, \underline{\phi}) < -1 \mid \underline{\tau}, \underline{b}, \underline{\phi}\right], \tag{5.11}$$

where $\eta_j^{(1)} \overset{\Delta}{=} \dfrac{\eta_j'^{(1)}}{\sqrt{\dfrac{P_1}{2}}\, T_c}$ has zero mean and variance $\dfrac{N_0 N}{2E_b^{(1)}}$.

The j^{th} sample of the interference is written as

$$I_j^{(1)}(\underline{b}, \underline{\tau}, \underline{\phi}) = \sum_{k=2}^{K} \sqrt{\epsilon_{k,1}}\, \frac{\cos\phi_k}{T_c} \left[B(j,m_k)\, a_j^{(1)}\, a_{j-m_k-1}^{(k)}\, \tau'_k + \hat{B}(j,m_k)\, a_j^{(1)}\, a_{j-m_k}^{(k)} (T_c - \tau'_k) \right], \tag{5.12}$$

where $\tau'_k \overset{\Delta}{=} \tau_k - m_k T_c$ and $m_k \overset{\Delta}{=} \lfloor \tau_k / T_c \rfloor$, $\epsilon_{k,1}$ is defined as the ratio of the powers $\dfrac{P_k}{P_1}$, $k=2,3,\ldots,K$, and the functions $B(\cdot,\cdot)$ and $\hat{B}(\cdot,\cdot)$ are defined to account for the possible partial overlap of adjacent bits; i.e.,

$$B(j,m_k) \overset{\Delta}{=} \begin{cases} b_{-1}^{(k)} & 0 \leq j \leq m_k \\ b_0^{(k)} & m_k+1 \leq j \leq N-1 \end{cases}$$

and

$$\hat{B}(j,m_k) \overset{\Delta}{=} \begin{cases} b_{-1}^{(k)} & 0 \leq j \leq m_k - 1 \\ b_0^{(k)} & m_k \leq j \leq N-1 \, . \end{cases} \tag{5.13}$$

Substituting (5.7) and (5.10) into (5.1), the average bit-error probability conditioned on the delays is written as

$$\overline{P}_e(\underline{\tau}) = \frac{1}{\pi^K\, 2^{2K-3}} \sum_{\underline{b}} \sum_{m=1}^{(N+1)/2} \int_0^{\pi} \left\{ \int_0^{\frac{\pi}{2}} \left[\prod_{j=0}^{N-1} R(u, p_j(\underline{\tau})) \cos\left[\sum_{j=0}^{N-1} \Delta(u, p_j(\underline{\tau}), 2m-1) \right] \right] du \right\} d\phi \,. \qquad (5.14)$$

Recall that the $\eta_j^{(1)}$'s are i.i.d. with a symmetric density function. Thus

$\Pr[\eta_j^{(1)} \geq 1 - I_j^{(1)}] = \Pr[\eta_j^{(1)} < -1 + I_j^{(1)}]$, and $E_{\underline{\phi},\underline{b}}[I_j^{(1)}(\underline{\phi}, \underline{\tau}, \underline{b})] = 0.$

Now since (5.7) and (5.10) are functions of $p_j(\underline{\tau})$ and $\hat{p}_j(\underline{\tau})$, respectively, and with the above properties we can express $\overline{P}_e(\underline{\tau})$ as in (5.14).

In the case that only two users share the channel (K=2), (5.14) can be simplified significantly by noting that the absolute value of $I_j^{(1)}$ takes only two forms:

$$I_j^{(1)}(\underline{b}, \underline{\tau}, \underline{\phi}) = \begin{cases} \pm\sqrt{\epsilon_{2,1}}\cos\phi_2\,(1-2d_2) \\ \pm\sqrt{\epsilon_{2,1}}\cos\phi_2, \end{cases} \qquad (5.15)$$

where $d_2 \overset{\Delta}{=} \tau'_2/T_c$. Now defining j_i, i=1,2,3,4 as the number of times in N tries that $I_j^{(1)}$ takes one of the four forms (e.g., $I_j^{(1)} = \sqrt{\epsilon_{2,1}}\cos\phi_2(1-2d_2)$) enables us to simplify (5.14) as

$$\overline{P}_e(\underline{\tau}) = \frac{1}{\pi^2\, 2^{2(K-2)}} \sum_{\underline{b}} \int_0^{\frac{\pi}{2}} \left\{ \int_0^{\frac{\pi}{2}} \left[\prod_{i=1}^{4} R(u, p(i))^{j_i} \sum_{m=1}^{(N+1)/2} \cos\left[\sum_{i=1}^{4} j_i \Delta(u, p(i), 2m-1) \right] \right] du \right\} d\phi \,, \qquad (5.16)$$

where $p(i) \overset{\Delta}{=} \Pr[\eta_j^{(1)} \geq 1 - I_j^{(1)}]$ when $I_j^{(1)}$ takes the i^{th} form i=1,2,3,4. Note that j_i, which is a function of the data bits of the other user, also depends on the signature sequences of the two users. An expression for $\overline{P}_e(\tau)$ similar to (5.16) can be obtained when more than two users share the channel. However, illustration of the final expression would be somewhat difficult.

5.2. The Combinatorial Method

We now derive an alternative expression for the average bit-error probability from the original definition (5.1) when only two users share the channel. Recall that the objective is to evaluate $\Pr[Y_N = m]$ for some odd m, where Y_N is the sum of N random variables $\hat{Z}_j^{(1)} \in \{-1,+1\}$. First we rewrite the four probabilities

$$P(1) \triangleq \Pr[\eta_j^{(1)} \geq 1 - \sqrt{\epsilon_{2,1}}\cos\phi_2]\,,$$

$$P(2) \triangleq \Pr[\eta_j^{(1)} \geq 1 + \sqrt{\epsilon_{2,1}}\cos\phi_2]\,,$$

$$P(3) \triangleq \Pr[\eta_j^{(1)} \geq 1 - \sqrt{\epsilon_{2,1}}\cos\phi_2(1-2d_2)]\,,$$

and $P(4) \triangleq \Pr[\eta_j^{(1)} \geq 1 + \sqrt{\epsilon_{2,1}}\cos\phi_2(1-2d_2)]$

which are conditioned on $\underline{\tau}$ and $\underline{\phi}$. The test statistic Y_N can take on any odd integral value between $-N$ and $+N$. Y_N equals an odd integer m only if the number of times in N tries $Z_j^{(1)}$ comes out non-negative is exactly $(N+m)/2$. The probability of each $Z_j^{(1)}$ being non-negative is equal to P(i) for some i, i=1,2,3,4, depending on the bits of the other user and the signature sequences. Recalling the definition for j_i, i=1,2,3,4 we can write

$$\Pr\left[Y_N = m \mid b_0^{(1)} = -1, \rho_I\right] = \sum_{n_4}\sum_{n_3}\sum_{n_2} \binom{j_1}{\frac{N+m}{2} - n_2 - n_3 - n_4} \binom{j_2}{n_2} \binom{j_3}{n_3} \binom{j_4}{n_4}$$

$$p(1)^{\frac{N+m}{2} - n_2 - n_3 - n_4} \left[1-p(1)\right]^{j_1 - \frac{N+m}{2} + n_2 + n_3 + n_4} p(2)^{n_2} \left[1-p(2)\right]^{j_2 - n_2} \tag{5.17}$$

$$p(3)^{n_3} \left[1-p(3)\right]^{j_3 - n_3} \; p(4)^{n_4} \left[1-p(4)\right]^{j_4 - n_4},$$

where $\binom{n}{k} \triangleq \frac{n!}{k!(n-k)!}$. In (5.14) the summations are in the ranges

$$\max\left[\frac{N+m}{2} - j_3 - j_2 - j_1, 0\right] \le n_4 \le \min\left[\frac{N+m}{2}, j_4\right],$$

$$\max\left[\frac{N+m}{2} - n_4 - j_2 - j_1, 0\right] \le n_3 \le \min\left[\frac{N+m}{2} - n_4, j_3\right],$$

and

$$\max\left[\frac{N+m}{2} - n_4 - n_3 - j_1, 0\right] \le n_2 \le \min\left[\frac{N+m}{2} - n_4 - n_3, j_2\right].$$

Now substituting (5.17) into (5.1), we can write

$$\overline{P}_e(\underline{\tau}) = \frac{1}{2\pi} \sum_{\underline{b}} \sum_{m=1}^{(N+1)/2} \int_0^{\frac{\pi}{2}} \Pr\left[Y_N = 2m-1 \mid b_0^{(1)} = -1, \underline{\tau}, \underline{b}, \underline{\phi}\right] d\phi \,. \tag{5.18}$$

As with the final simplification (5.16) in Section 5.1, the analytical result obtained here can be generalized to the case when more than two users share the channel, albeit it is somewhat messy to do so.

5.3. Numerical Results

The numerical examples presented here are aimed at showing that, for fixed SNR, improvement in DS/SSMA performance can be obtained by using hard-limiting correlation receivers in place of linear correlators when the tails of the noise distribution are heavy. Thus, we compute average bit-error probabilities given a relative time delay between users for impulsive noise sources interfering with the two-user binary PSK direct-sequence SSMA system considered in the previous chapter.

This analysis is first carried out with fixed signal-to-noise ratio and equal signal power assumption for all the users. With the variance of the random variable $\eta_j^{(1)}$ held constant, changes in error probability are analyzed for different noise distributions. Tables 5.1 and 5.2 contain the average bit-error probabilities for Gaussian, Laplacian, and ε-mixture examples with a typical delay. Results indicate significant improvement in performance by using hard-limiting correlation receivers in place of linear correlators for more impulsive noise channels. Moreover, this improvement becomes more visible as the length of the signature sequences used by the channel subscribers increases. Figures 5.1 and 5.2 support this conclusion over a range of SNR values for the ε-mixture example. An interesting conclusion here is that, unlike the linear correlation receiver, degradation due to the other user is no longer uniform in the range of signal-to-noise values. Thus, as the SNR increases, the hard-limiter apparently is not as effective in separating the two users as the linear correlation receiver is. However, this deficiency is outweighed by the improvement against the impulsive noise as long as the channel noise is significant (i.e., the SNR is moderate).

Finally, we consider the near-far effects in the performance of hard-limiting DS/SSMA correlation receivers. In this context, we vary the signal power of the second user with respect to the first user's power level and then compute the error probability. For Gaussian, Laplacian, and ε-mixture channels, Tables 5.3 and 5.4 are generated for values of $\epsilon_{2,1}$ less than unity and greater than unity, respectively. Additionally, Figure 5.3 is drawn to illustrate the performance variations over a range of values for $\epsilon_{2,1}$. Comparing these results with the analogous ones for the linear correlator of

TABLE 5.1. ERROR PROBABILITIES OF HARD-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; ε-MIXTURE CHANNEL, SNR=8.0 dB, TWO USERS WITH EQUAL POWERS AND TYPICAL TIME DELAYS.

ε, γ^2	K	N=31	N=63
0.0,1 *	1	2.19×10^{-3}	2.25×10^{-3}
	2	4.63×10^{-3}	2.51×10^{-3}
0.1,10	1	1.59×10^{-4}	1.47×10^{-4}
	2	9.53×10^{-4}	2.14×10^{-4}
0.01,100	1	3.71×10^{-5}	3.71×10^{-5}
	2	3.90×10^{-4}	5.77×10^{-5}
0.1,100	1	1.86×10^{-12}	2.68×10^{-15}
	2	2.59×10^{-6}	2.06×10^{-10}

* Gaussian noise

TABLE 5.2. ERROR PROBABILITY OF THE HARD-LIMITING CORRELATION RECEIVER IN THE BINARY PSK DS/SSMA SYSTEM; GAUSSIAN AND LAPLACIAN CHANNELS, SNR = 8.0, TWO USERS WITH EQUAL POWERS AND TYPICAL TIME DELAYS.

	GAUSSIAN		LAPLACIAN
N=31	K=1	2.19×10^{-3}	1.05×10^{-4}
	K=2	4.63×10^{-3}	8.67×10^{-4}
N=63	K=1	2.25×10^{-3}	3.57×10^{-5}
	K=2	2.51×10^{-3}	1.02×10^{-4}
N=127	K=1	2.27×10^{-3}	1.32×10^{-5}
	K=2	2.77×10^{-3}	5.41×10^{-5}

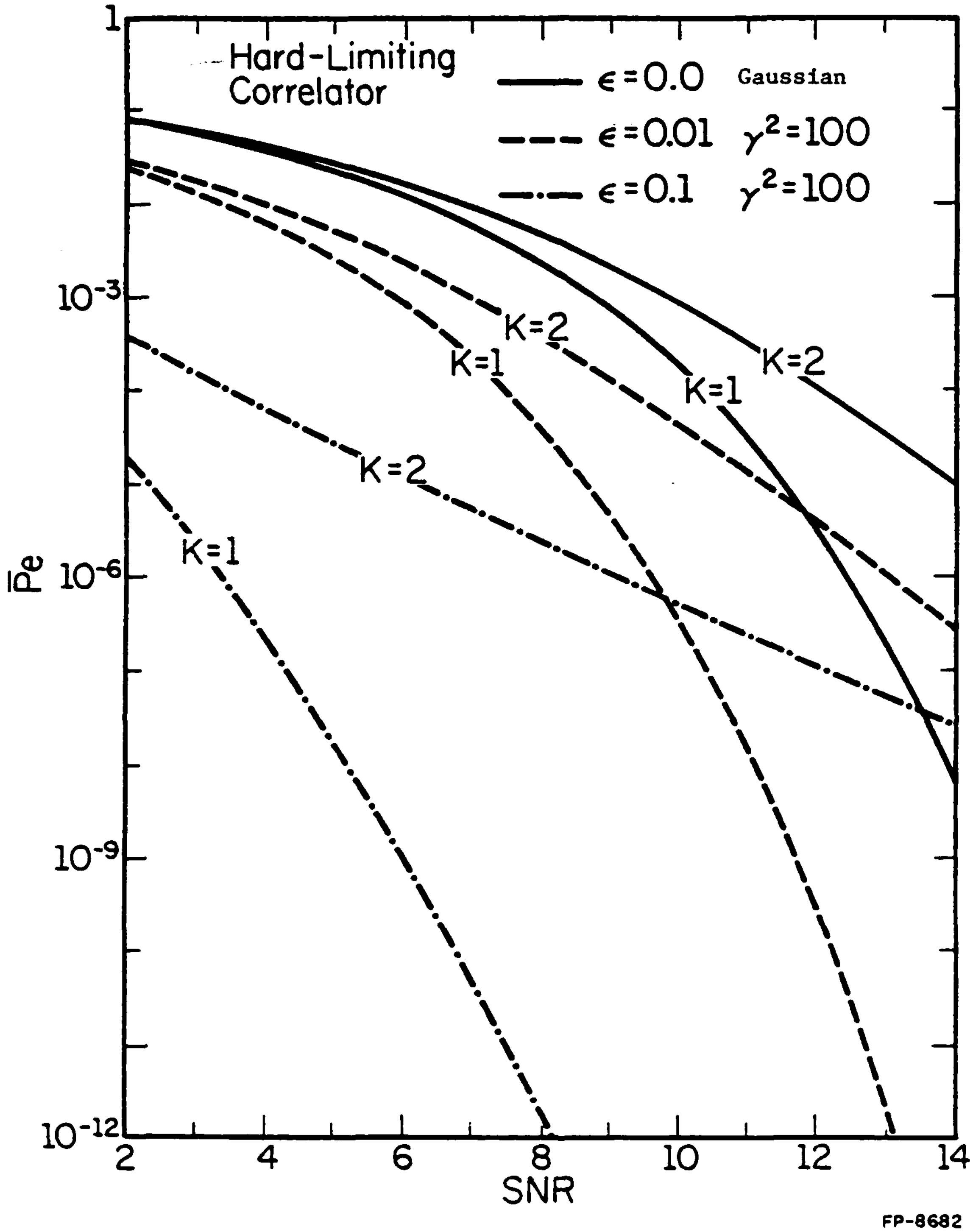

Figure 5.1. Error probability of the hard-limiting correlation receiver in ε-mixture channels, N=31, two users with equal power and typical time delays.

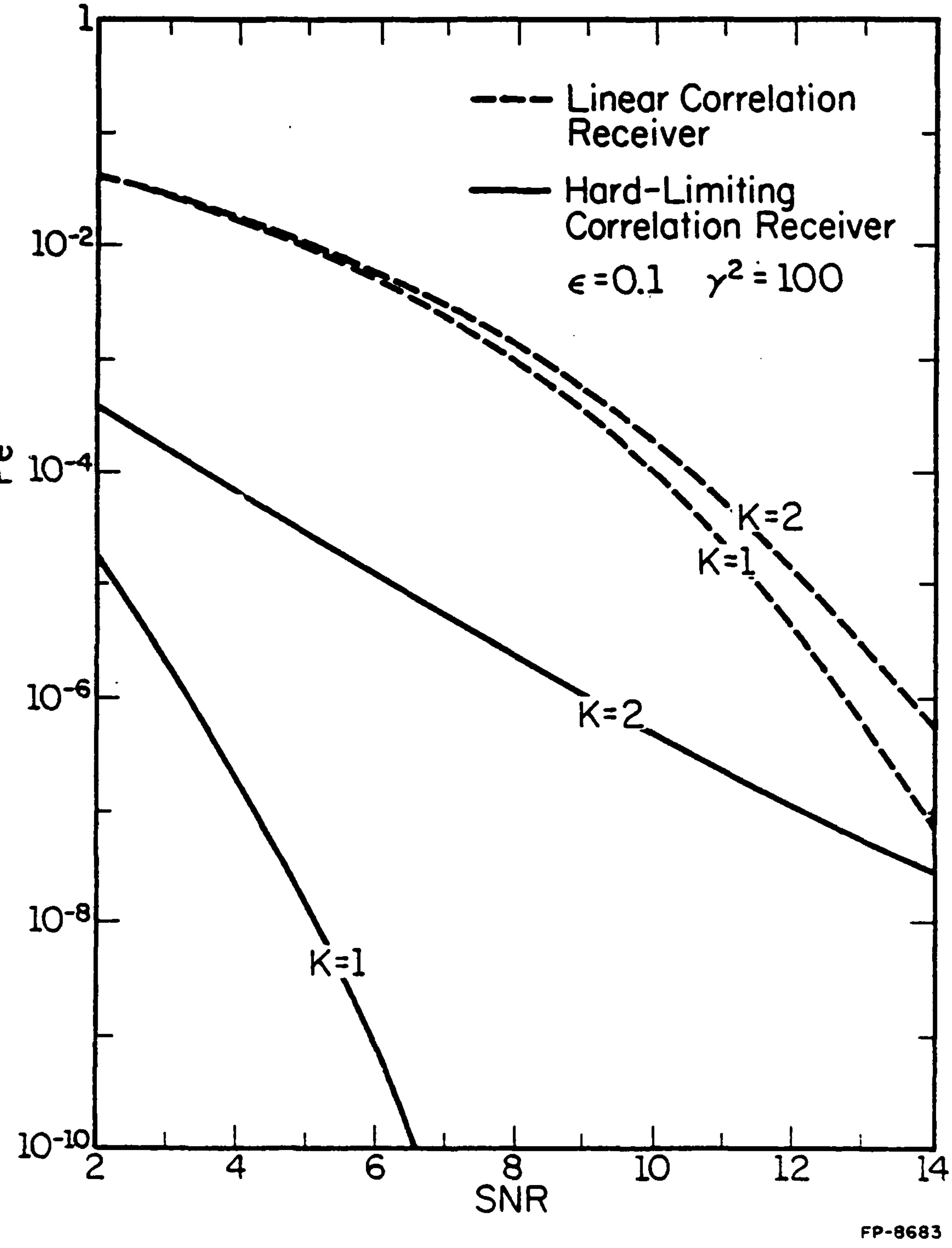

Figure 5.2. Error probability of linear and hard-limiting correlation receives in ε-mixture channels, N=31, two users with equal power and typical time delays.

TABLE 5.3. ERROR PROBABILITY OF HARD-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND IMPULSIVE CHANNELS, SNR=8.0 dB, N=31, K=2, TWO USERS WITH UNEQUAL POWERS AND TYPICAL TIME DELAYS ($\epsilon_{2,1} = P_2/P_1 \leq 1$).

DISTRIBUTION	$\epsilon_{2,1} = -6.02dB$	$\epsilon_{2,1} = -3.01dB$	$\epsilon_{2,1} = -1.25dB$	$\epsilon_{2,1} = 0.0dB$
GAUSSIAN	2.69×10^{-3}	3.27×10^{-3}	3.91×10^{-3}	4.63×10^{-3}
LAPLACIAN	1.77×10^{-4}	3.01×10^{-4}	5.13×10^{-4}	8.67×10^{-4}
ε-MIXTURE				
ε=0.01, γ^2=100	7.57×10^{-5}	1.40×10^{-4}	2.41×10^{-4}	3.90×10^{-4}
ε=0.1, γ^2=100	2.68×10^{-10}	1.43×10^{-8}	2.74×10^{-7}	2.59×10^{-6}

TABLE 5.4. ERROR PROBABILITY OF HARD-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND IMPULSIVE CHANNELS, SNR=8.0 dB, N=31, K=2, TWO USERS WITH UNEQUAL POWERS AND TYPICAL TIME DELAYS ($\epsilon_{2,1} = P_2/P_1 \geq 1$).

DISTRIBUTION	$\epsilon_{2,1} = 0.0dB$	$\epsilon_{2,1} = 3.01dB$	$\epsilon_{2,1} = 4.77dB$
GAUSSIAN	4.63×10^{-3}	8.30×10^{-3}	1.32×10^{-2}
LAPLACIAN	8.67×10^{-4}	4.05×10^{-3}	9.58×10^{-3}
ε-MIXTURE			
ε=0.01, γ^2=100	3.90×10^{-4}	1.74×10^{-3}	4.93×10^{-3}
ε=0.1, γ^2=100	2.59×10^{-6}	4.21×10^{-4}	3.85×10^{-3}

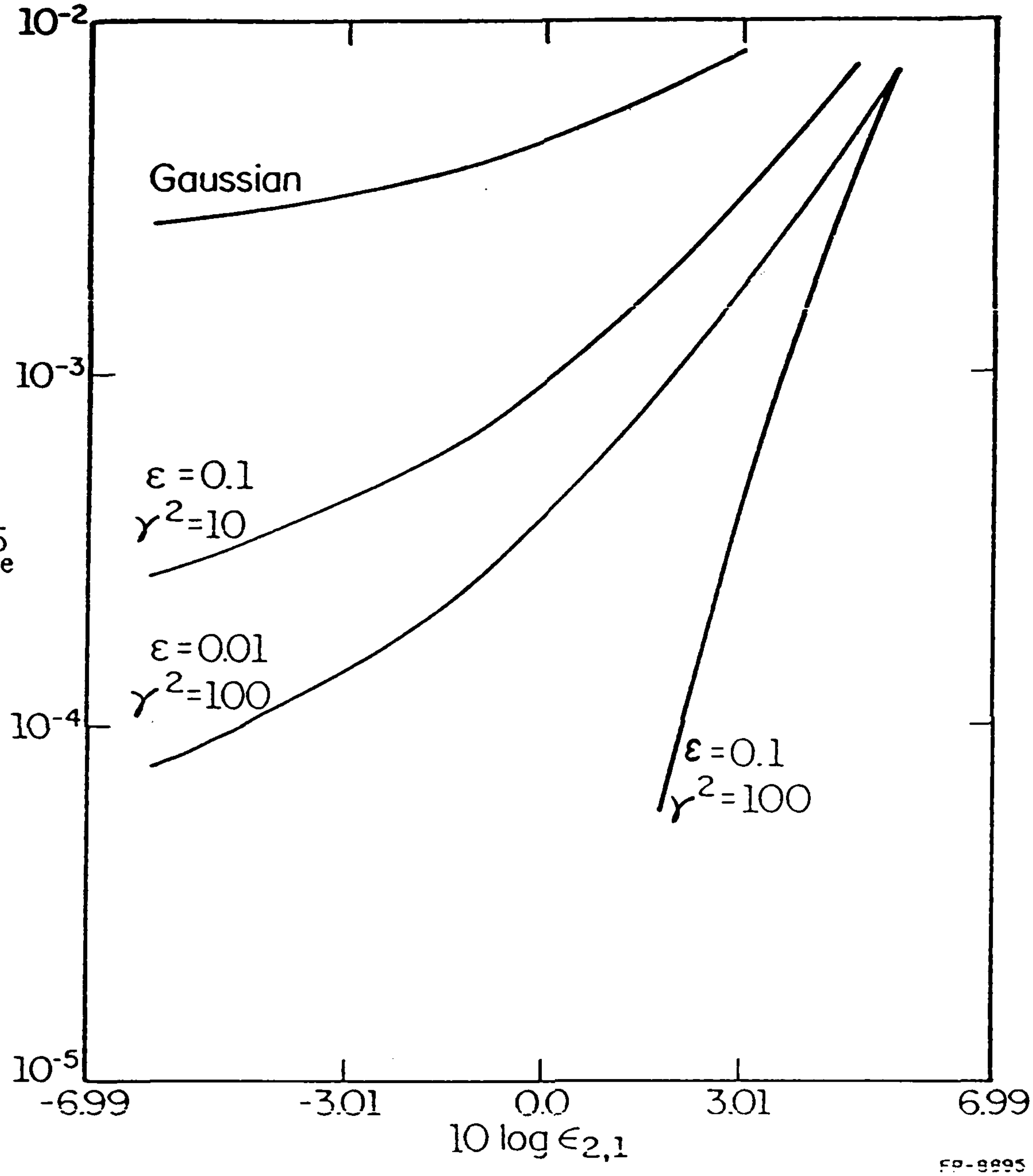

Figure 5.3. Near-far effects in the error probability of the hard-limiting correlation receiver in ε-mixture channels, SNR (of user one) =8 dB, K=2, N=31.

Chapter 4 we see that, as the other user's relative power increases, there is a point beyond which the linear correlator is superior to the hard-limiting correlator. This again is due to the fact that the linear correlator appears to be more effective than the hard-limiter is in dealing with the MA interference. So when the MA interference dominates the impulsive interference, the linear correlator is better. However, this crossover point is higher for more impulsive models and in most cases occurs beyond the desirable range of error probabilities. Thus, we can conclude that the hard-limiter correlator is still desirable in a two-user impulsive channel provided the second user is not dominant.

Summarizing this chapter, we have considered the performance of hard-limiting correlation receiver in a typical two-user DS/SSMA system operating in impulsive noise. We have seen that in most cases, the hard-limiter outperforms the linear correlator, particularly when the MA interference is dominated by the impulsive channel noise. Similar to the analysis for the linear receiver, we experienced a rapid growth in the computational effort required to compute $\overline{P}_e$ for larger N. The problem of approximating $\overline{P}_e$ for large values of N will be discussed in Chapter 7.

CHAPTER 6

LINEAR CORRELATION RECEIVERS IN IMPULSIVE CHANNELS: APPROXIMATIONS

In earlier chapters, expressions for average bit-error probability were obtained for both linear and hard-limiting correlation receivers operating in the presence of multi-user noise and non-Gaussian impulsive noise. The computational effort in finding $\bar{P}_e$ grows rapidly as N, the number of chips per data bit, and K, the number of users sharing the channel, increase. In fact, in our previous analysis we have been limited by computational considerations to considering exclusively the case K=2. In this chapter, we consider further the performance of linear correlation receivers in the prescribed environment. Maintaining the same system model, we introduce an approximation, an upper bound and a lower bound for the average probability of error that allow for easier computation with large N. The approximation to the performance of the linear correlation receiver is based on a Taylor series expansion of the average error probability. Similar ideas have been used in [11,12] and [43]. We will also show that this approximation is exact asymptotically in N. Furthermore, we consider the moment-space bounding technique proposed in [44-45], and [17] to provide upper and lower bounds on the average probability of error in this case.

6.1. Taylor Series Approximation

In this section, we obtain an approximation for the error probability of the linear correlation receiver by truncating a Taylor series expansion. It is easy to show that with the test statistic for the linear correlator, the average bit-error probability for the receiver can be written as (see also [12])

$$\bar{P}_e = E_{\underline{\tau},\underline{\phi},\underline{b}} \left\{ \Psi_N \left[\frac{1+I_1}{\sqrt{\frac{N_0}{2E_b^{(1)}}}} \right] \right\}, \tag{6.1}$$

where $\Psi_N(a) \triangleq \Pr\left[\sum_{j=0}^{N-1} \eta''_j > a \right]$ with zero mean samples of noise η''_j normalized to have variance $\frac{1}{N}$.

The multiple-access interference in (6.1) is defined as

$$I_1(\underline{b}, \underline{\tau}, \underline{\phi}) = \sum_{k=2}^{K} \sqrt{\epsilon_{k,1}} \frac{\cos\phi_k}{T} \left\{ b_{-1}^{(k)} R_{k,1}(\tau_k) + b_0^{(k)} \hat{R}_{k,1}(\tau_k) \right\}, \tag{6.2}$$

where $\epsilon_{k,1}$ the ratio of the powers $\frac{P_k}{P_1}$, $k = 2,3,\ldots,K$, and the continuous-time partial cross-correlation functions $R_{k,1}$ and $\hat{R}_{k,1}$ are given in (4.8) and (4.9), respectively. Following [11,12] and [43] we expand the function Ψ_N in (6.1) in Taylor series about a function of the single-user SNR, $\alpha \triangleq \sqrt{\frac{2E_b^{(1)}}{N_0}}$. Then if we take the expectation of each term (assuming this interchange is permissible) we will have the following:

$$\bar{P}_e = \Psi_N(\alpha) + \sum_{m=1}^{\infty} \frac{\alpha^{2m}}{2m!} E\left\{ I_1^{2m} \right\} \Psi_N^{(2m)}(\alpha) , \tag{6.3}$$

where $\Psi_N^{(m)}$ is the m^{th} derivative of Ψ_N. Note that, if Ψ_N is analytic the series in (6.3) converges uniformly in α because the interference is bounded. In (6.3) the first term of the series is the error probability in the absence of multiple-access noise. The rest of the series is a function of signal-to-noise ratio, the shape of the noise distribution, and the even moments of the multiple-access noise.

The moments $E[\,I_1^{2m}\,]$ can be evaluated by means of a recursion (see [11,12] and [43]). We first define the random variables

$$W_{q,1} \triangleq \frac{\sqrt{\epsilon_{q,1}}}{T}\left\{ b_{-1}^{(q)} R_{q,1}(\tau_q) + b_0^{(q)} \hat{R}_{q,1}(\tau_q) \right\}, \quad q = 2, 3, \ldots, K. \tag{6.4}$$

Let $I_{q,1}$ be the partial sum of the interference of users 2, 3, . . . , and q. This quantity can be written as

$$I_{q,1} = \sum_{k=2}^{q} W_{k,1} \cos \phi_k = I_{q-1} + W_{q,1} \cos \phi_q \ , \quad q = 3,4,\ldots,K, \tag{6.5}$$

with $I_{2,1} = W_{2,1} \cos \phi_2$. Note that $I_1 = I_{K,1}$, and since ϕ_q and $W_{q,1}$ are independent we can write

$$E[\,I_{q,1}^{2m}\,] = \sum_{i=0}^{m} \binom{2m}{2i} \binom{2(m-i)}{m-i} 2^{-2(m-i)} E[\,I_{q-1,1}^{2i}\,]\, E[\,W_{q,1}^{2(m-i)}\,]\,, \quad q = 3,\ldots,K. \tag{6.6}$$

(Notice that due to symmetry of the multiple-access interference, the odd moments of $I_{q,1}$ are zero.) For computing the moments of the multiple-access interference we must first evaluate $E[\,W_{q,1}^{2m}\,]$. To compute these moments, we use a modified version of the characteristic function of $I_1(\underline{\tau}, \underline{\phi}, \underline{b})$ given in (4.14). This yields

$$E[\,W_{q,1}^{2m}\,] = \frac{1}{2N} \sum_{n=0}^{N-1} [\, f_m(n, \sqrt{\epsilon_{q,1}}\, \theta_{q,1}) + f_m(n, \sqrt{\epsilon_{q,1}}\, \hat{\theta}_{q,1})\,]\ , \tag{6.7}$$

where

$$f_m(n, y) \triangleq \frac{\left[\frac{y(n+1)}{N}\right]^{2m+1} - \left[\frac{y(n)}{N}\right]^{2m+1}}{(2m+1)\frac{[\,y(n+1) - y(n)\,]}{N}} \tag{6.8}$$

for $y(n) \neq y(n+1)$ and $f_m(n, y) \triangleq [\frac{y(n)}{N}]^{2m}$ if $y(n) = y(n+1)$.

Referring to expression (6.3), our proposition is that if the signature sequences are chosen correctly, then when N is large the effect of higher degree moments of $I_1(\underline{b}, \underline{\tau}, \underline{\phi})$ in (6.3) becomes negligible (note that in most cases we can safely assume that I_1 is bounded from above by 1). Therefore, only a few terms of the infinite series should be needed to obtain a small truncation error for $\overline{P}_e$, and thus in order to evaluate the estimate for $\overline{P}_e$, we primarily need to study the distribution function Ψ_N and its lower-order derivatives.

Recall that our first-order non-Gaussian channel model describes the noise samples η''_j. Let us start with the Gaussian distribution; of course, the sum of N *independent* zero mean Gaussian random variables each with variance $\frac{1}{N}$ is a Gaussian random variable with zero mean and unit variance. That is,

$$\Psi_N(\alpha) = Q(\alpha) \triangleq \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \frac{e^{-x^2}}{2} dx \ . \tag{6.9}$$

We can write the nth derivative of Q in terms of the Hermite polynominals $H_m(x)$ which are defined by (see [25]) $H_0(x) = 1, H_1(x) = 2x$, and

$$H_{m+1}(x) = 2x\, H_m(x) - 2m\, H_{m-1}(x)\ ; \ \ m \geq 1. \tag{6.10}$$

The resulting expression for $\overline{P}_e$ is

$$\bar{P}_e = Q(\alpha) + \frac{e^{\frac{-\alpha^2}{2}}}{\sqrt{\pi}} \sum_{m=1}^{\infty} \frac{\left[\frac{\alpha}{\sqrt{2}}\right]^{2m}}{2m!} H_{2m-1}\left(\frac{\alpha}{\sqrt{2}}\right) E\left[I_I^{2m} \right] . \tag{6.11}$$

The convergence of this infinite series and an upper bound on the truncation error that results when the series is approximated by the sum of a finite number of terms are considered in [12], [43], and also [11].

For the ε-mixture example, recall from Chapter 3 that the probability distribution function of the sum of N independent random variables, each having a density given in (2.7), can be written as

$$\Psi_N(a) = \sum_{i=0}^{N} \binom{N}{i} (1-\varepsilon)^i \varepsilon^{N-i} Q(\beta_i a) , \tag{6.12}$$

where $\beta_i \triangleq \sqrt{\frac{N[(1-\varepsilon)+\varepsilon\gamma^2]}{i+(N-i)\gamma^2}}$. The resulting expression for $\bar{P}_e$ for the ε–mixture case is thus

$$\bar{P}_e = \sum_{i=0}^{N} \binom{N}{i} (1-\varepsilon)^i \varepsilon^{N-i} \left\{ Q(\beta_i\alpha) + \frac{e^{\frac{-(\beta_i\alpha)^2}{2}}}{\sqrt{\pi}} \sum_{m=1}^{\infty} \frac{(\beta_i \frac{\alpha}{\sqrt{2}})^{2m}}{2m!} H_{2m-1}(\beta_i \frac{\alpha}{\sqrt{2}}) E\left[I_I^{2m} \right] \right\}, \tag{6.13}$$

and the truncation properties will be similar to those for the Gaussian case discussed in [11] and [12].

For this case, by using only two terms of the infinite series in (6.11) and (6.13) we obtain an estimate of $\bar{P}_e$ for the linear correlator. Typical results showing the accuracy of this approximation are depicted in Figure 6.1. These approximations are computed for Gaussian and ε-mixture examples with fixed signal-to-noise ratio (SNR = 8 dB) for some values of spreading sequence lengths N. As we anticipated, the estimate results in values for the error probability closer to the exact ones as N grows. For the N=31, K=6 Gaussian case, this method is less accurate than for K=2. However, for

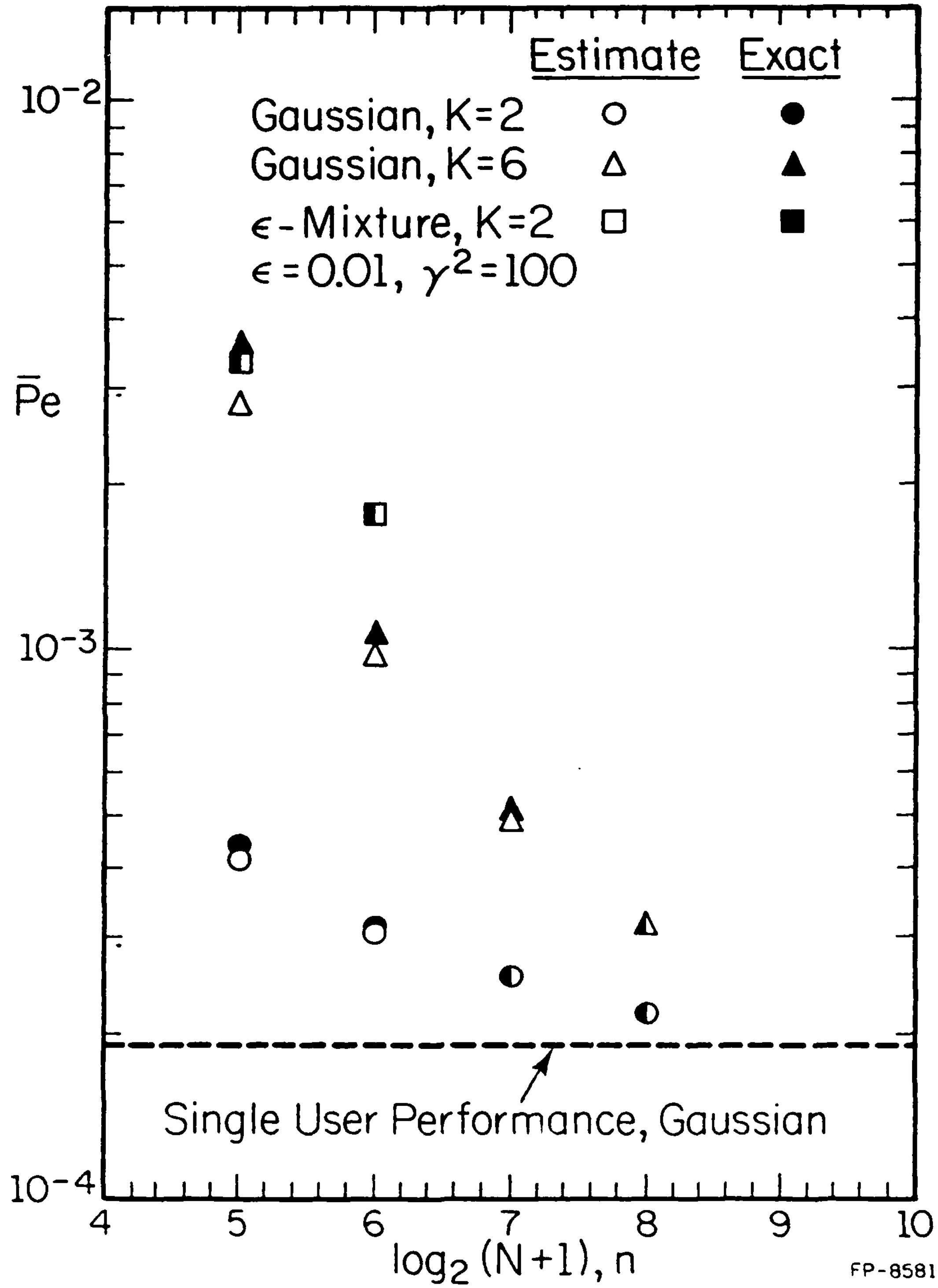

Figure 6.1. Taylor series lower bound and exact error probabilities for linear correlators in the presence of Gaussian and ε-mixture noise, SNR = 8.0 dB.

highly impulsive examples, the approximation is very accurate even for small N. This is most likely due to the small values of the higher-order derivatives of heavily tailed distributions at α. Considering the above finding, the Taylor series approximation appears to be very appropriate for estimating the error probability for the impulsive non-Gaussian channels. Using this technique, we approximate the error probability of the linear correlation receiver in ε-mixture channels for three-and six-user systems. The results are contained in Table 6.1 with signal-to-noise ratio value of 8 dB and signature-sequence lengths N=31 and N=63. As we learned from the above discussion, the estimates in Table 6.1 must be more accurate for the cases with larger values of N and more impulsive noise channels. We also see that when the multi-user interference becomes more significant (e.g., K = 6), the channel noise represents a smaller fraction of the total noise. Consequently, the performance of the linear correlation receiver in these impulsive channels becomes comparable to the performance of that receiver in a Gaussian channel under similar conditions.

6.2. Asymptotic Analysis

Equation (6.3) yields a useful approximation to the average bit-error probability for linear correlation receivers in impulsive noise for large N. Now we consider the limitation of the system as N grows without bound. Recall from the single-user analysis of Chapter 3 that in considering asymptotics we wish to keep the energy per data bit, the length of the data period T, the noise power, and consequently, the signal-to-noise ratio, constant as N changes. The statistic for the binary decision for the linear correlator is

$$Y_N = \sum_{j=0}^{N-1} \left(\eta_j'^{(1)} + I_j'^{(1)} + \sqrt{\frac{P_1}{2}} T_c b_0^{(1)} \right) \tag{6.14}$$

where $\eta_j'^{(1)}$'s , $j = 0, 1, \ldots, N-1$ are the noise components of the sampled signal with zero means and variances $\frac{N_0 T_c}{4}$. The $I_j'^{(1)}$'s are samples of the multiple-access interference and are defined as

TABLE 6.1. TAYLOR SERIES APPROXIMATION FOR ERROR PROBABILITIES OF LINEAR CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND ε-MIXTURE CHANNEL, SNR=8.0 dB.

ε, γ^2	K	N=31	N=63
0.0,1 *	3	8.40×10^{-4}	4.29×10^{-4}
	6	3.59×10^{-3}	1.09×10^{-3}
0.1,10	3	1.14×10^{-3}	5.55×10^{-4}
	6	3.84×10^{-3}	1.24×10^{-3}
0.01,100	3	4.01×10^{-3}	1.95×10^{-3}
	6	6.56×10^{-3}	2.78×10^{-3}
0.1,100	3	1.85×10^{-3}	8.76×10^{-3}
	6	4.13×10^{-3}	1.62×10^{-3}

* Gaussian noise

$$I_j'^{(1)} \triangleq \sqrt{\frac{P_1}{2}} T_c I_j^{(1)}$$

where $I_j^{(1)}$ is given in (5.12). Referring to (6.14), conditioned on the multiple-access parameters $\rho_I = (\underline{\tau}, \underline{\phi}, \underline{b})$, the test statistic Y_N is seen to be the sum of N independent random variables. Thus, the asymptotic error probability of the linear correlation receiver can be obtained by applying the central limit theorem to the sum, as follows:

With the test statistic as given in (6.14), the asymptotic error probability is given as

$$\overline{P}_e = \lim_{N \to +\infty} E_{\underline{\tau},\underline{\phi},\underline{b}} \left\{ \tfrac{1}{2} \Pr(Y_N \geq 0 \mid b_0^{(1)} = -1, \rho_I) + \tfrac{1}{2} \Pr(Y_N < 0 \mid b_0^{(1)} = +1, \rho_I) \right\}. \quad (6.15)$$

This expression can be simplified using the assumed symmetry as

$$\overline{P}_e = \lim_{N \to +\infty} E_{\underline{\tau},\underline{\phi},\underline{b}} \left\{ \Pr(Y_N < 0 \mid b_0^{(1)} = +1, \rho_I) \right\}. \quad (6.16)$$

Now the term inside the expectation on the right-hand side of (6.16) can be written as

$$\Pr(Y_N < 0 \mid b_0^{(1)} = +1, \rho_I) =$$

$$\Pr\left[\frac{Y_N - E\{Y_N \mid b_0^{(1)} = +1, \rho_I\}}{\sqrt{\mathrm{Var}(Y_N \mid b_0^{(1)} = +1, \rho_I)}} < \frac{-E\{Y_N \mid b_0^{(1)} = +1, \rho_I\}}{\sqrt{\mathrm{Var}(Y_N \mid b_0^{(1)} = +1, \rho_I)}} \;\middle|\; b_0^{(1)} = +1, \rho_I \right]. \quad (6.17)$$

Note that the conditional mean of the test statistic Y_N is given by

$$E\left\{Y_N \mid b_0^{(1)}=+1,\rho_I\right\}=\sqrt{\frac{P_1}{2}}\,T_c\,[\sum_{j=0}^{N-1}(\,I_j^{(1)}+1\,)\,]\;, \tag{6.18a}$$

where $I_j^{(1)}$ is defined in (5.12) and relates to the total multiple-access interference as $I_1 = 1/N \sum_{j=0}^{N-1} I_j^{(1)}$.

Similarly, the conditional variance is given by

$$\mathrm{Var}\left[Y_N \mid b_0^{(1)}=+1,\rho_I\right]=\frac{N\,N_0\,T_c}{4}. \tag{6.18b}$$

Thus, assuming that $\lim_{N\to+\infty} I_1\,(\underline{\tau},\underline{\phi},\underline{b})$ exists almost surely, the central limit theorem implies that

$$\lim_{N\to+\infty} \Pr\left[Y_N<0 \mid b_0^{(1)}=+1,\rho_I\right]=Q\left[\alpha[\,1+\lim_{N\to+\infty} I_1(\underline{b},\underline{\tau},\underline{\phi})\,]\right]\;(\mathrm{a.s.}). \tag{6.19}$$

Therefore, since $Q \le 1$, the bounded convergence theorem implies that the asymptotic multi-user error probability of the linear correlation receiver can be written as

$$\overline{P}_e=E_{\underline{\tau},\underline{\phi},\underline{b}}\left\{Q\left[\alpha[\,1+\lim_{N\to+\infty} I_1(\underline{b},\underline{\tau},\underline{\phi})\,]\right]\right\}, \tag{6.20}$$

where $I_1(\underline{b},\underline{\tau},\underline{\phi})$ and α are as defined in (6.2) and (6.3).

From (6.20) we see that single-user performance in a multi-user environment is achieved asymptotically by the linear correlation receiver if the following asymptotic condition is satisfied:

$$\lim_{N\to+\infty} I_1(\underline{b},\underline{\tau},\underline{\phi})=0 \text{ almost surely.} \tag{6.21}$$

Furthermore, in Chapter 8, we will show that (6.21) is satisfied if

$$\lim_{N\to+\infty} \max_{m} \frac{1}{N} |\theta_{k,1}(m)| = 0 \text{ and } \lim_{N\to+\infty} \max_{m} \frac{1}{N} |\hat{\theta}_{k,1}(m)| = 0 \ , \ k = 2,3,\ldots,K. \tag{6.22}$$

The existence of binary sequences satisfying these conditions follows by applying (6.22) to infinite sequences proposed by Schneider and Orr [34]. Schneider and Orr show the existence of a sequence set whose autocorrelation and cross-correlation functions approach ideal behavior (6.21) with increasing N. Moreover, they prove that the cardinality of such sets of infinitely long sequences grows with N. Thus, at least one large set of binary sequences having asymptotically ideal correlation properties exists. However, by utilizing the sequences in [34], the convergence of (6.21) to zero is very slow. This induces a limit on the number of subscribers to the channel with single-user performance. From the discussion in [34], it follows that one can find at least $N^{1/4}$ sequences such that (6.21) is satisfied.

Figures 3.8 and 3.9 in Chapter 3 contain the asymptotic bit-error probability for the linear correlation receiver under the assumption that (6.21) is satisfied, since the multi-user impulsive noise performance in this case is the same as in the Gaussian single-user case. Note that the above asymptotic results also indicate that the approximate error probability obtained from truncating the infinite sum in (6.3) is asymptotically exact for appropriately chosen sequence sets.

6.3. Moment-Space Bounds

In this section, we apply the moment-space bounding technique to bound the average bit-error probability of the linear correlation receiver in non-Gaussian impulsive noise. This method, which was developed by Yao in [44] and [45], is based on a result from the theory of games. To present the application of the bound to our problem, we first state a simple version of an isomorphism theorem in moment spaces which provides relations among arbitrary moments of a random variable.

Note from (6.1) that our main objective here is to evaluate the mean of a function of the random variable $I_1(\underline{\tau}, \underline{\phi}, \underline{b})$. Although a closed-form expression for the probability distribution function F_I of this random variable is not available, we know that its support is confined to a finite closed interval $\Xi = [-D, D]$. Referring to (6.2) and from [28] we have the result that for rectangular chip waveforms

$$D = \frac{1}{N}\sum_{k=2}^{K} \max_{m}\left\{ |C_{k,1}(m)| + |C_{k,1}(m-N)| \ : \ 0 \le m \le N \right\}. \tag{6.23}$$

Now let χ_1 and χ_2 be a pair of continuous functions defined on the interval Ξ. The generalized moments of the random variable I_1 induced by the functions $\chi_i(I_1)$ are

$$m_i = \int_{\Xi} \chi_i(I_1)\, dF_I = E_{\underline{\tau}, \underline{\phi}, \underline{b}}\left\{ \chi_i(I_1) \right\}, \ i = 1, 2. \tag{6.24}$$

For a given pair of continuous functions (χ_1, χ_2), we denote the moment space M as

$$M = \left\{ m = (m_1, m_2) \in R^2 \mid m_i = \int_{\Xi} \chi_i(I)\, dF_I, i=1,2, F_I \in P(\Xi) \right\}, \tag{6.25}$$

where $P(\Xi)$ is the set of probability distribution functions defined on $\Xi = [-D, D]$. It is easy to see that M is a closed, bounded, and convex set in R^2. The isomorphism theorem states that if we plot $\chi_1(y)$ versus $\chi_2(y)$ in R^2 for y ranging over Ξ, then the convex hull H of the resulting plot is the moment-space M (i.e. $H = M$).

The application of this result to bounding the error probability of the linear correlation receiver is given as follows. Let the function χ_2 be defined by the expression inside the curly brackets in (6.1); that is,

$$\chi_2(y) = \Psi_N [\, \alpha(1+y) \,] \ . \tag{6.26}$$

Now let $\chi_1(y)$ be some continuous function of y whose generalized moment m_1 may be readily evaluated. If this function is carefully chosen so that the resulting curve $(\chi_1(y), \chi_2(y))$ has a thin convex hull over the range Ξ, then we can use our knowledge of m_1 to obtain tight upper and lower bounds on m_2, since $H = M$ and, for this χ_2,

$$m_2 = \int_{\Xi} \chi_2(I_1)\, dF_{I_1}. \tag{6.27}$$

We would like to choose χ_1 so that the convex hull of the curve of χ_2 versus χ_1 is thin. However, in choosing χ_1, we must trade off the thickness of H with the evaluability of m_1. A useful family of candidate functions for χ_1 consists of exponentials of the form

$$\chi_1(y) = \exp\{h[1+y]\} \ , \tag{6.28}$$

where h is a real parameter which can be chosen to tighten the bounds. For each choice of the kernel we need to evaluate the corresponding moment. Referring to (6.2), we note that

$$m_1 = e^h \prod_{k=2}^{K} \left[E\left\{ \exp[\, h I_{k,1}(b_k, \tau_k, \phi_k) \,] \right\} \right] \ , \tag{6.29}$$

where $I_{k,1}$ is the multiple-access interference corresponding to the k^{th} user. The moment-generating function of the random variable $I_{k,1}$ appearing in (6.29) can be written as

$$E\left\{\exp\left[\, h\, I_{k,1}(b_k, \tau_k, \phi_k)\,\right]\right\} = \frac{1}{2N}\sum_{m=0}^{N-1}\int_0^1 \left[I_0\!\left(h\sqrt{\epsilon_{k,1}}\left[\frac{1-t}{N}\theta_{k,1}(m) + \frac{t}{N}\theta_{k,1}(m+1)\right]\right)\right.$$

$$\left. + I_0\!\left(h\sqrt{\epsilon_{k,1}}\left[\frac{1-t}{N}\hat{\theta}_{k,1}(m) + \frac{t}{N}\hat{\theta}_{k,1}(m+1)\right]\right)\right] dt\ , \qquad (6.30)$$

where I_0 is the modified Bessel function of order zero. The moment of $\chi_1(I_1)$ is obtained by substituting (6.30) into (6.28).

Having chosen χ_1 to be of the form (6.28), we use a graphical technique to evaluate the upper and lower bounds on the error probability, in which we plot $\chi_1(y)$ versus $\chi_2(y)$ over the range Ξ and complete the convex hull *H* graphically. Since we can compute m_1 via (6.25) the upper and lower bounds on the error probability can easily be read from the graph by drawing a line perpendicular to the axis corresponding to $\chi_1(y)$ from point m_1.

We also must search for the best value of h graphically. In order to help find the best h, we plot $\chi_1(y)$ versus both $\chi_2(y)$ and $\chi'_2(y)$, where the latter is defined as

$$\chi'_2(y) = \Psi_N\left[\,\alpha(1-y)\,\right], \qquad (6.31)$$

and look for a close fit. Then, we define *H'* as the convex hull of the plot of χ'_2 versus χ_1, and m'_2 as the moment of $\chi'_2(I_1)$. Note that the intersection of *H* and *H'* contains moments corresponding to symmetric probability distribution functions in $P(\Xi)$. Moreover, the convex set *H* for h is the same as the set *H'* for $-$h. Therefore, we only need to consider positive values of h. (Note that $m_2 = m'_2$.)

For plotting $\chi_2(y)$ and $\chi'_2(y)$ versus $\chi_1(y)$, we need to evaluate Ψ_N for values of y in the range Ξ. For the ε-mixture example, we use (6.12) and draw the curve for various distribution functions in this model. Figures 6.2 through 6.7 depict the sets *H* and *H'* for these distributions and for N = 31

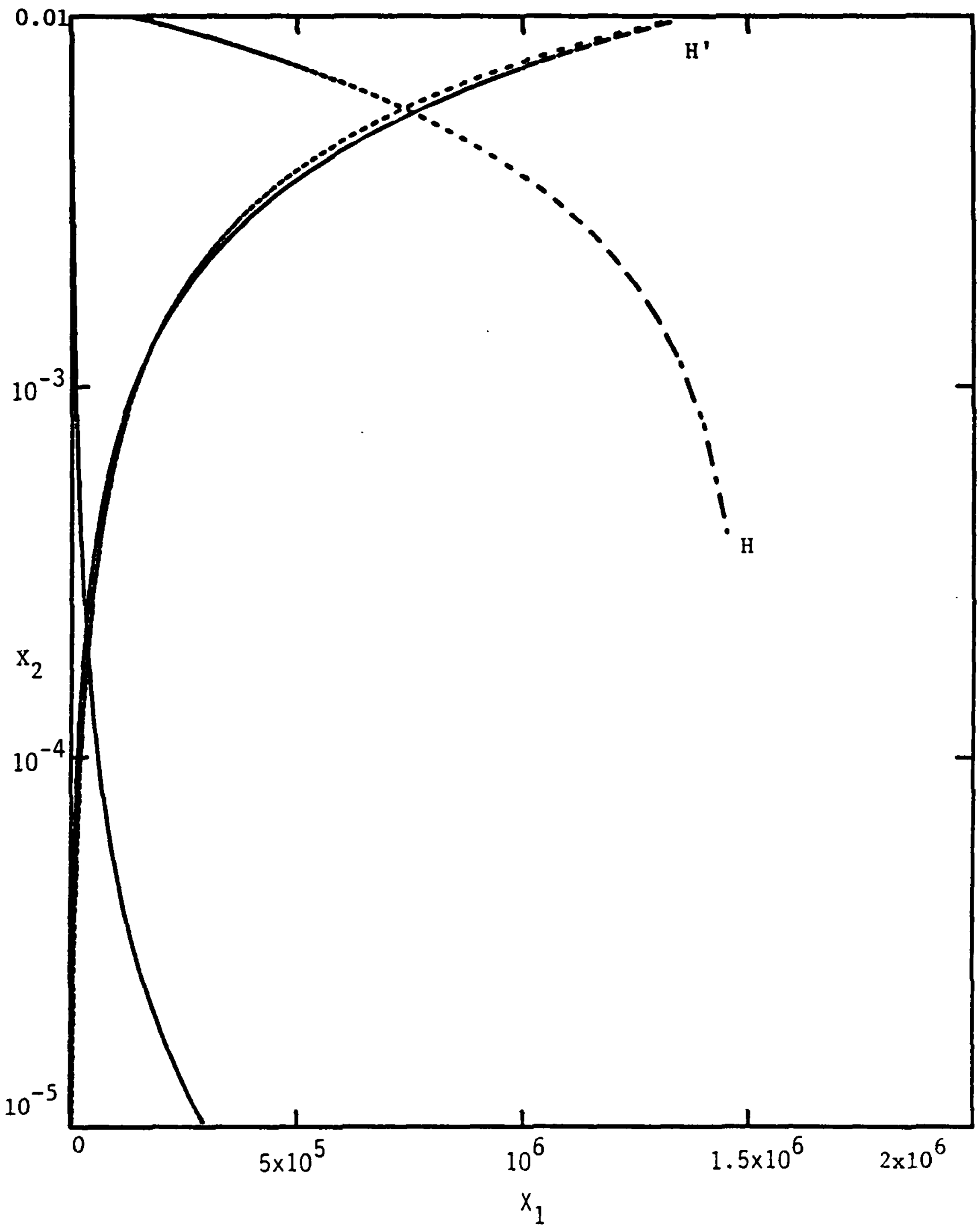

Figure 6.2. The convex sets H and H' for moment-space bounding for Gaussian example with SNR = 8 dB, and N=31, h = 10.5.

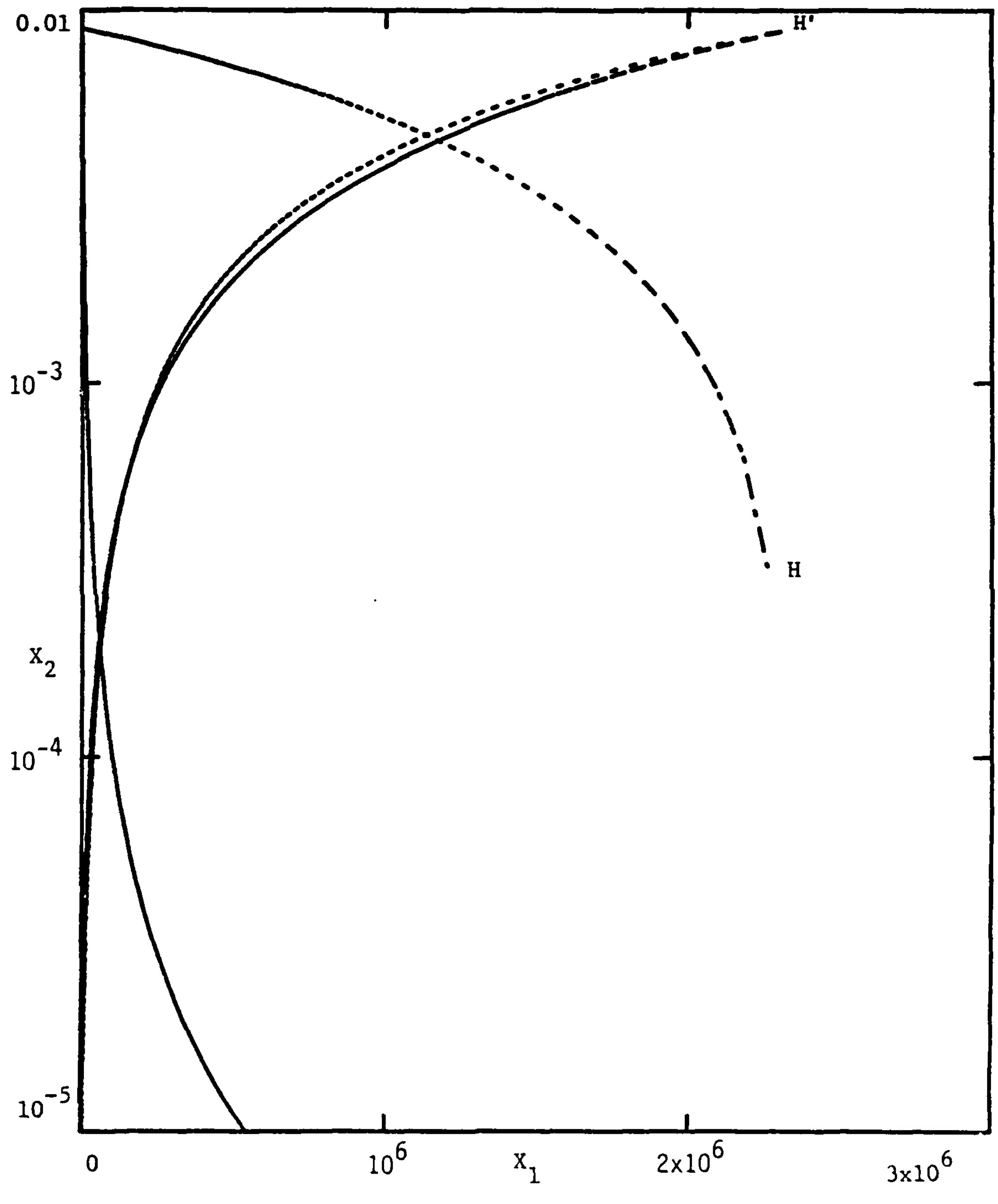

Figure 6.3. The convex sets H and H' for moment-space bounding for the Gaussian example with SNR = 8 dB, and N=63, h = 11.0.

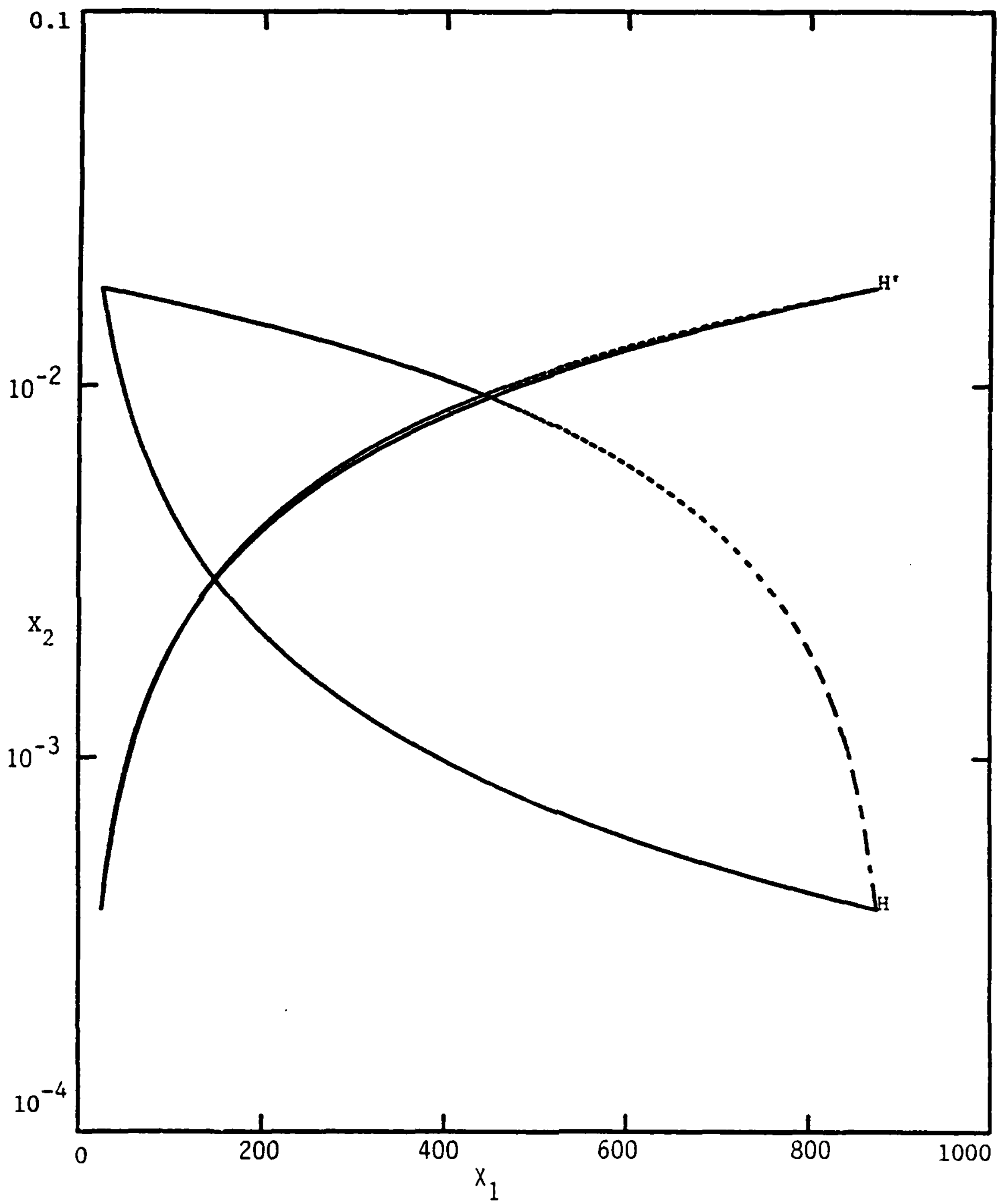

Figure 6.4. The convex sets H and H' for moment-space bounding for ε-mixture example with ε=0.01, γ^2=100, SNR = 8 dB, and N=31, h = 5.0.

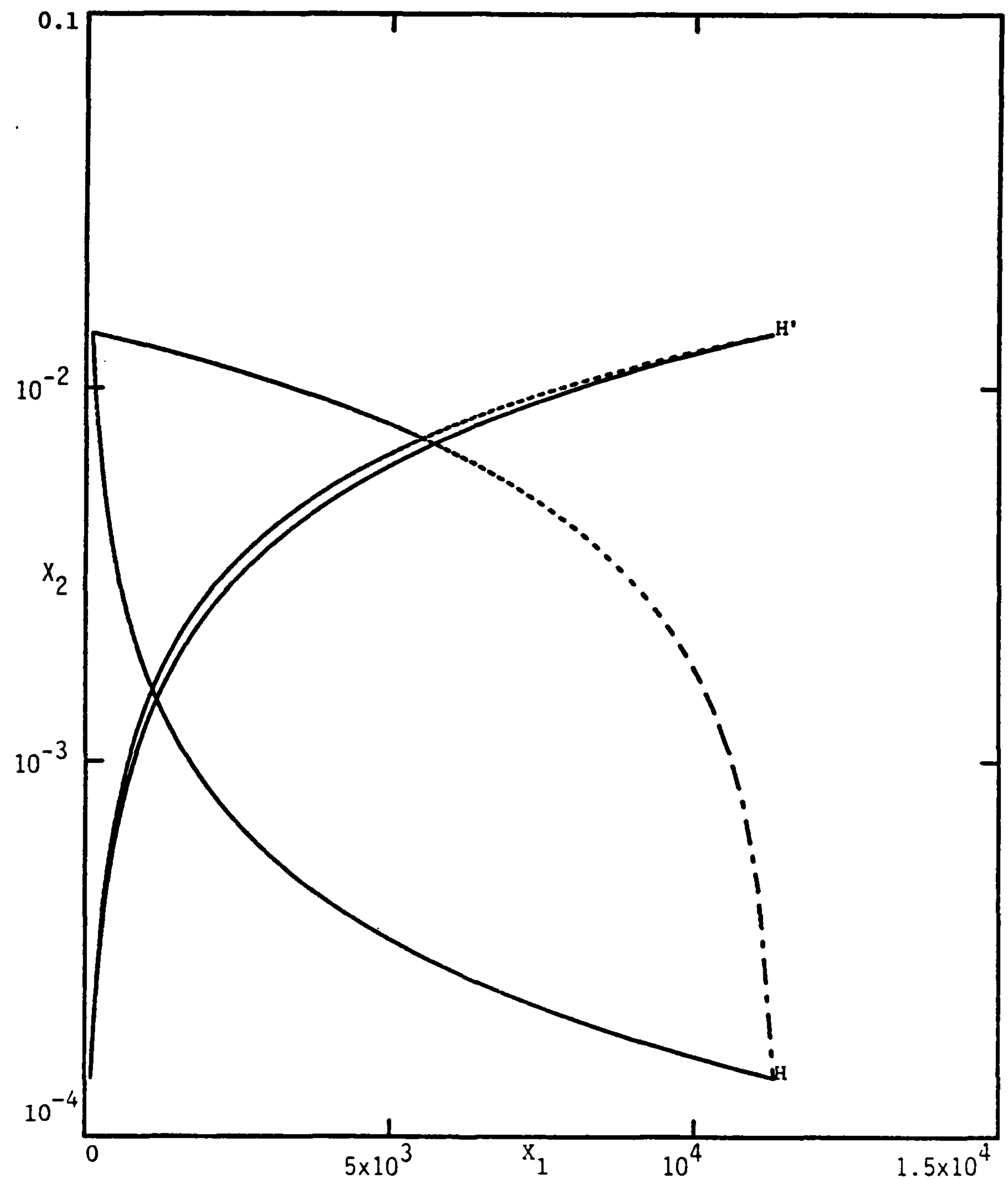

Figure 6.5. The convex sets H and H' for moment-space bounding for ε-mixture example with ε=0.01, γ^2=100, SNR = 8 dB, and N=63, h = 7.0.

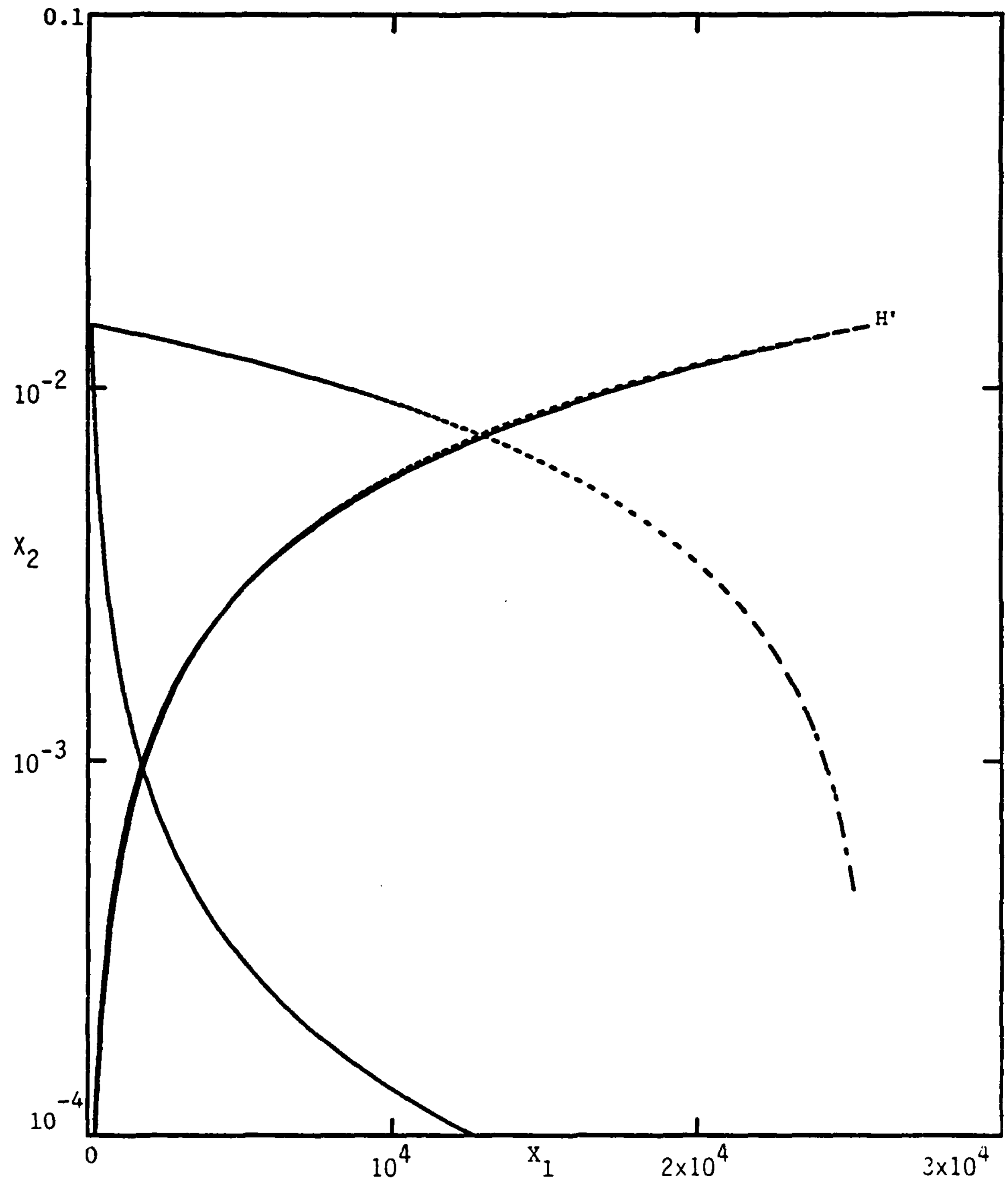

Figure 6.6. The convex sets H and H' for moment-space bounding for ε-mixture example with ε=0.1, γ^2=100, SNR = 8 dB, and N=31, h = 7.5.

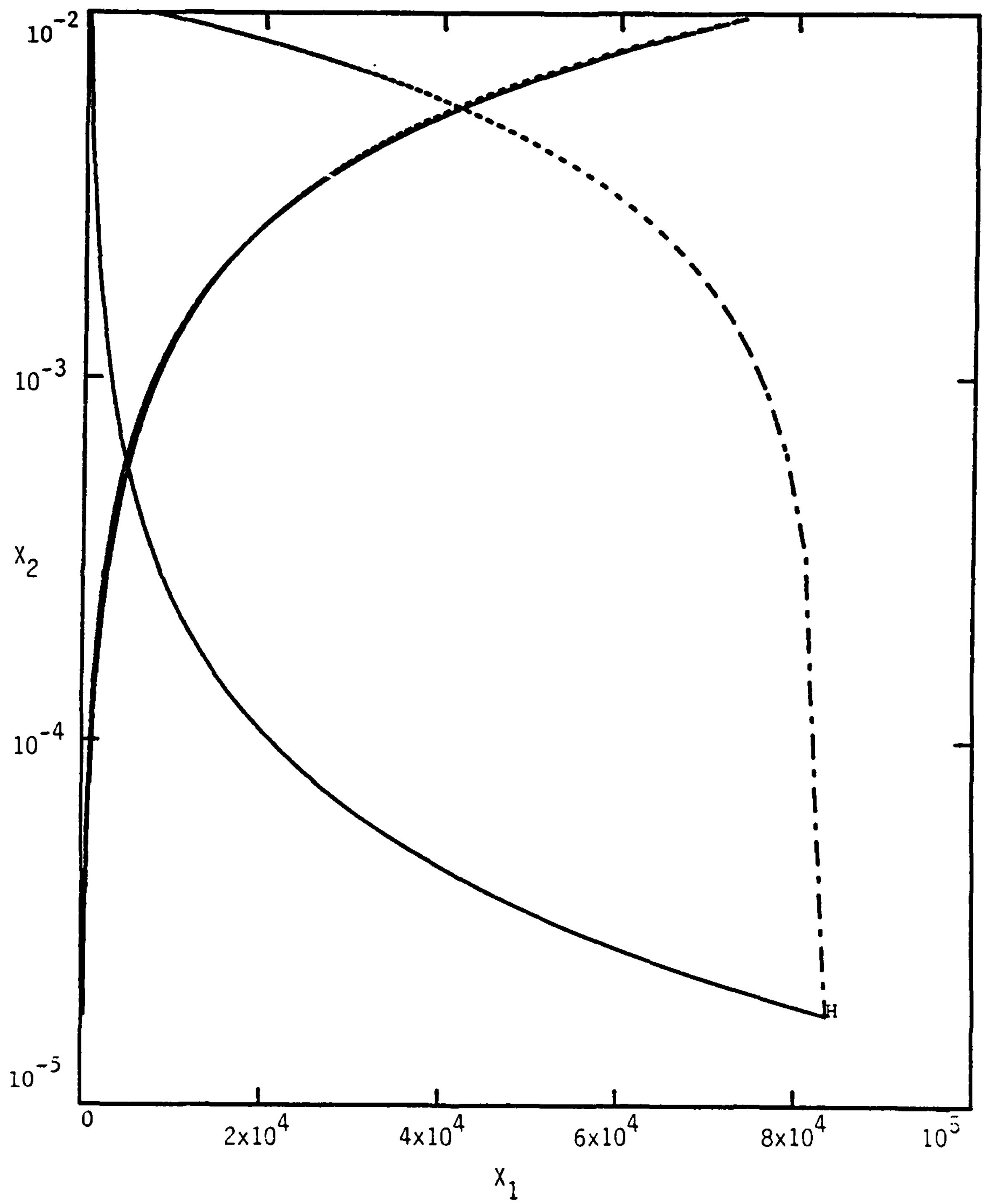

Figure 6.7. The convex sets H and H' for moment-space bounding for ε-mixture example with ε=0.1, γ^2=100, SNR = 8 dB, and N=63, h = 8.5.

and 63 with the same DS/SSMA system configurations considered in previous examples. Notice that in these plots the convexity of these sets is not apparent since the vertical axes are in logarithmic scale and the horizontal axes are in linear scale. We apply the moment-space bound to two and six-user DS/SSMA systems in impulsive channels. The results are summarized in Tables 6.2 and 6.3. As expected, for a fixed number of users K, as N increases, D decreases and the bounds tend to become tighter. On the other hand, with fixed N, as the number of users increases, the bounds become looser. Similar to what happened for the Taylor-series approximation, the moment-space bound results in better performance approximation for more impulsive channels. A possible explanation for this is that the exponential kernel in (6.25) better approximates $\chi_2(y)$ given in (6.23) for impulsive channels than it does for the Gaussian case. (Note from Table 6.1 that the Taylor series approximation lies within the moment-space bounds in all cases common to both sets of results.)

6.4. Summary

In this chapter we have used two techniques to estimate the average error probability of the linear correlation receiver in non-Gaussian impulsive channels. These methods were introduced as an alternative to the exact computation proposed in Chapter 4 when N or K is large. Particularly for impulsive channels, both techniques appear to estimate the performance closely. The principal difficulty that arises in applying these methods is that the probability distribution function of the sum of N non-Gaussian random variables has to be evaluated for some points near the signal-to-noise ratio. However, this problem is essentially that of single-user analysis, and for the ε-mixture case it is trivial.

TABLE 6.2. MOMENT-SPACE UPPER AND LOWER ERROR BOUNDS FOR LINEAR CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND ε-MIXTURE CHANNELS, SNR=8.0 dB, N=31.

DISTRIBUTION	K	P_e^L	P_e	P_e^U
GAUSSIAN	2	3.93×10^{-4}	4.16×10^{-4}	4.64×10^{-4}
	6	1.44×10^{-4}	3.48×10^{-3}	8.58×10^{-3}
ε-MIXTURE ε=0.01, γ^2=100	2	3.41×10^{-3}	3.46×10^{-3}	3.47×10^{-3}
	6	4.10×10^{-3}		8.80×10^{-3}
ε=0.1, γ^2=100	2	1.31×10^{-3}	1.31×10^{-3}	1.35×10^{-3}
	6	1.51×10^{-3}		8.80×10^{-3}

TABLE 6.3. MOMENT-SPACE UPPER AND LOWER ERROR BOUNDS FOR LINEAR CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND ε-MIXTURE CHANNELS, SNR=8.0 dB, N=63.

DISTRIBUTION	K	P_e^L	P_e	P_e^U
GAUSSIAN	2	2.96×10^{-4}	3.06×10^{-4}	3.24×10^{-4}
	6	3.91×10^{-4}		1.33×10^{-3}
ε-MIXTURE ε=0.01, γ^2=100	2	1.72×10^{-3}	1.76×10^{-3}	1.77×10^{-3}
	6	2.13×10^{-3}		3.39×10^{-3}
ε=0.1, γ^2=100	2	7.19×10^{-3}	7.19×10^{-3}	7.22×10^{-3}
	6	8.97×10^{-4}		2.48×10^{-3}

CHAPTER 7

HARD-LIMITING CORRELATION RECEIVERS IN IMPULSIVE CHANNELS: APPROXIMATIONS

One conclusion from the hard-limiting correlation receiver analysis in Chapter 5 is that computing the exact expression for $\bar{P}_e$ can be very time consuming for large N. For this reason, when signature sequences of length more than 31 are used, bounds and approximations are needed to replace the exact computation of $\bar{P}_e$. In this chapter we propose an approximation, an upper bound, and a lower bound for the multi-user error probability of the hard-limiting correlation receiver in impulsive noise. The lower bound uses the idea of truncating a finite sum of positive numbers in the expression for the error probability (5.18); the approximation is based on estimating the probability distribution function of the test statistic Y_N; and the upper bound uses a Chernoff bounding technique. Also in this chapter we analyze the hard-limiting correlation receiver's performance asymptotically in N.

7.1. A Truncated Series Lower Bound on $\bar{P}_e$

Our first bound on the error probability of the hard-limiting correlation receiver uses the property that $\Pr\left[Y_N = 2m-1 \mid b_0^{(1)} = -1, \rho_I\right]$ decreases sharply as m increases to (N + 1)/2. The event $\{ Y_N = m \}$ has very small probability for large values of m when conditioned on $b_0^{(1)} = -1$, because Y_N taking some positive value m means the output of the hard-limiter was +1 exactly $(N+m)/2$ times, an event which becomes much less likely as m increases. Therefore, in computing $\bar{P}_e$ via (see also (5.18))

$$\bar{P}_e(\underline{\tau}) = \sum_{m=1}^{(N+1)/2} \frac{1}{2\pi} \sum_{\underline{b}} \int_0^{\frac{\pi}{2}} \Pr\left[Y_N = 2m-1 \mid b_0^{(1)} = -1, \underline{\tau}, \underline{b}, \underline{\phi}\right] d\phi , \tag{7.1}$$

we can obtain a reasonably tight lower bound by adding only the first few terms of the sum of odd m's from 1 to N. The accuracy of this approximation is easily controlled by including more terms in the computation, noting the tradeoff with the cost. As an example, we compute a lower bound by including only $\lfloor \log_2(N+1) \rfloor - 1$ terms in the summation in (7.1) instead of $(N+1)/2$. For the ε-mixture and Laplacian examples, we examine the tightness of this lower bound of $\bar{P}_e(\underline{\tau})$ for the hard-limiting correlation receiver. Typical results showing the accuracy of this approximation are depicted in Figure 7.1 and Tables 7.1 and 7.2. (Also shown are approximate values to be discussed below.) These results are obtained for the same DS/SSMA system considered in examples in Chapter 5. Examples are carried out with lengths of signature sequences equal to 31 and 63. Tables 7.1 and 7.2 correspond to SNR = 8 dB and SNR = 4 dB, respectively. As we see from these tables and the figure, the bound is generally tight. Moreover, for more impulsive examples, which are the most interesting ones, the lower bound is extremely tight.

7.2. Binomial Approximation

The above bound is still computationally expensive, although much less so than exact computation. Thus, simpler estimates of $\bar{P}_e$ are of interest. One such estimate can be based on approximating the distribution function of the test statistic Y_N, which is given as

$$Y_N = \sum_{j=0}^{N-1} \hat{Z}_j^{(1)} = \sum_{j=0}^{N-1} \operatorname{sgn} \left[\eta'^{(1)}_j + I'^{(1)}_j + \sqrt{\frac{P_1}{2}}\, T_c b_0^{(1)} \right]. \tag{7.2}$$

The error probability for the hard-limiting correlation receiver can be written in a form slightly different from (7.1), namely,

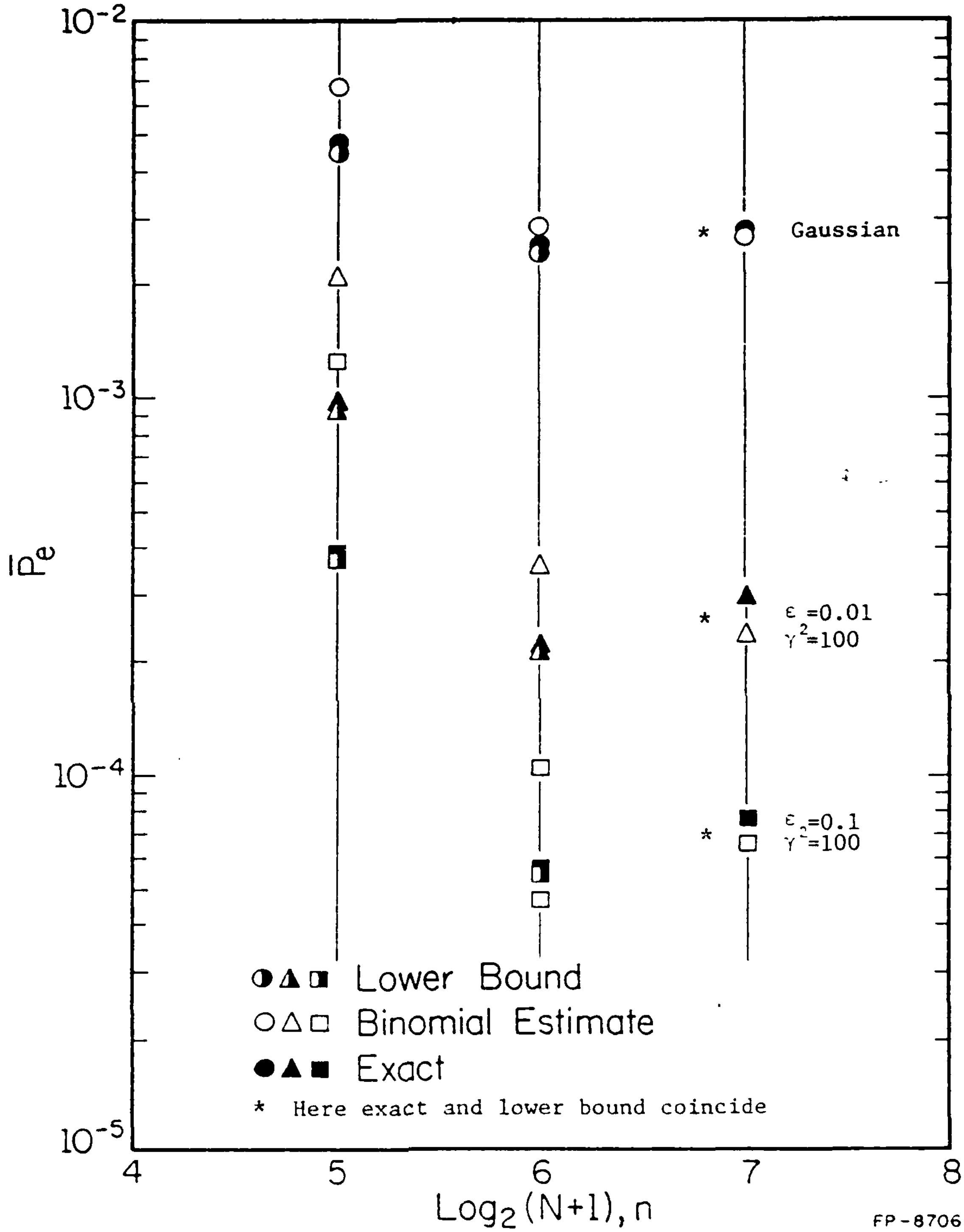

Figure 7.1. Truncated series lower bound and binomial approximation for the error probability of hard-limiting correlation receiver in ε-mixture channels, SNR = 8 dB.

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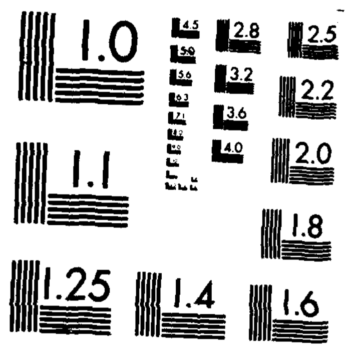

MICROCOPY RESOLUTION TEST CHART
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TABLE 7.1. TRUNCATED SERIES LOWER BOUND AND BINOMIAL APPROXIMATION FOR THE ERROR PROBABILITY OF HARD-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND IMPULSIVE CHANNELS, SNR=8.0 dB, K=2 AND TYPICAL TIME DELAYS.

DISTRIBUTION	N	P_e^L	$P_e \approx$	P_e
GAUSSIAN	31	4.51×10^{-3}	6.92×10^{-3}	4.63×10^{-3}
	63	2.45×10^{-3}	2.93×10^{-3}	2.51×10^{-3}
LAPLACIAN	31	8.55×10^{-4}	1.77×10^{-3}	8.67×10^{-4}
	63	1.01×10^{-4}	1.63×10^{-4}	1.02×10^{-4}
ε-MIXTURE $\varepsilon=0.01, \gamma^2=100$	31	3.87×10^{-4}	1.16×10^{-3}	3.90×10^{-4}
	63	5.73×10^{-5}	1.05×10^{-4}	5.77×10^{-5}
$\varepsilon=0.1, \gamma^2=100$	31	2.58×10^{-6}	4.54×10^{-5}	2.59×10^{-6}
	63	2.06×10^{-10}	1.68×10^{-8}	2.06×10^{-10}

TABLE 7.2. TRUNCATED SERIES LOWER BOUND AND BINOMIAL APPROXIMATION FOR THE ERROR PROBABILITY OF HARD-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND IMPULSIVE CHANNELS, SNR=4.0 dB, K=2 AND TYPICAL TIME DELAYS.

DISTRIBUTION	N	P_e^L	$P_e \approx$	P_e
GAUSSIAN	31	3.89×10^{-2}	4.52×10^{-2}	4.14×10^{-2}
	63	3.45×10^{-2}	3.81×10^{-2}	3.68×10^{-2}
LAPLACIAN	31	9.77×10^{-3}	1.26×10^{-2}	1.01×10^{-2}
	63	4.70×10^{-3}	5.37×10^{-3}	4.84×10^{-3}
ε-MIXTURE $\varepsilon=0.01, \gamma^2=100$	31	9.39×10^{-3}	1.27×10^{-2}	9.72×10^{-3}
	63	6.33×10^{-3}	7.21×10^{-3}	6.54×10^{-3}
$\varepsilon=0.1, \gamma^2=100$	31	6.63×10^{-5}	3.42×10^{-4}	6.66×10^{-5}
	63	7.60×10^{-7}	3.78×10^{-6}	7.61×10^{-7}

$$\bar{P}_e(\underline{\tau}) = E_{\underline{b},\underline{\phi}} \left\{ \Pr \left[Y_N < 0 \mid b_0 = +1 , \rho_I^{(1)} \right] \right\} . \tag{7.3}$$

Note that, conditioned on the multiple-access parameters, the test statistic $(Y_N+N)/2$ is multinomially distributed. Here we approximate this distribution by a binomial distribution for large N because the approximation is asymptotically exact as N grows without bound, and the binomial distribution has many attractive analytical properties. Thus, we approximate (7.3) as

$$\bar{P}_e(\underline{\tau}) \approx E_{\underline{b},\underline{\phi}} \left\{ \sum_{j=\frac{N+1}{2}}^{N} \binom{N}{j} q_I^{\,j} (1-q_I)^{N-j} \right\} , \tag{7.4}$$

where q_I is the "success" probability associated with the approximating binomial distribution. To choose q_I we equate the mean of the binomial approximation,

$$E\left\{ Y_N | b_0^{(1)}=+1, \rho_I \right\} = N [1-2 q_I], \tag{7.5}$$

with the actual mean of the test statistic,

$$E\left\{ Y_N \mid b_0^{(1)}=+1, \rho_I \right\} = N - 2 \sum_{j=0}^{N-1} \Pr \left[\eta_j^{(1)} \geq 1 + I_j^{(1)} \right] . \tag{7.6}$$

For the ε-mixture example (which includes Gaussian noise as a special case) q_I is given as

$$q_I = \frac{1}{N} \sum_{j=0}^{N-1} \left[(1-\varepsilon)\, Q \left(\frac{1+I_j^{(1)}}{\sigma_n} \right) + \varepsilon\, Q \left(\frac{1+I_j^{(1)}}{\gamma \sigma_n} \right) \right], \tag{7.7}$$

where $\sigma_n'^2 \triangleq \frac{N_0 N}{(1-\varepsilon+\varepsilon\gamma^2)\, 2E_b^{(1)}}$. For the Laplacian example, q_I is computed from the following expression

$$q_I = \frac{1}{2N} \sum_{j=0}^{N-1} \exp\left\{ -\sqrt{\frac{2}{\sigma^2}} \left[1 + I_j^{(1)} \right] \right\}, \tag{7.8}$$

where $\sigma^2 \triangleq \frac{N_0 N}{2 E_b^{(1)}}$, and we have assumed that $| I_j^{(1)} | \leq 1$, which holds for example when $K = 2$ or when $\epsilon_{k,1} = 1$ for all k. When K=2, (7.7) and (7.8) can be simplified by noting that $| I_j^{(1)} |$ takes only two forms as in (5.15). Substituting (7.7) or (7.8) into (7.4) yields the binomial approximation for the error probability $\bar{P}_e(\underline{\tau})$ for the hard-limiting correlation receiver.

Figure 7.1 and Tables 7.1 and 7.2 contain some numerical results that compare this binomial approximation to the above bound and to the exact values of $\bar{P}_e(\underline{\tau})$ for different values of N. (A DS/SSMA system identical to the one used in previous chapters is considered.) The approximation is evaluated for various parameters of ε-mixture noise and also for Laplacian noise. Considering the simplicity of the binomial approximation, the results in Figure 7.1 and Tables 7.1 and 7.2, particularly for large N, indicate that (7.4) is a reasonably close estimate of $\bar{P}_e(\underline{\tau})$. In Section 7.4. we will see that if signature sequences are chosen appropriately, the binomial approximation is asymptotically exact.

7.3. Chernoff Bound

In this section, we examine the tightness of the Chernoff bound when applied to the hard-limiting correlation receiver problem. Saltzberg in [30] and Lugannani in [15] used this method for intersymbol interference error bounding, which is similar to the problem of interest here. They found the Chernoff bound to be superior to existing upper bounds in intersymbol interference studies. However, our investigation (not reported in this thesis) showed that applying the Chernoff bound to the multi-user communication system in impulsive noise with the *linear* correlation receiver results in a loose bound for error probability. For the linear case the moment-space bounds are better. However, the moment-space technique cannot be applied directly to the hard-limiting receiver, and so despite the possibility of achieving a loose bound, we pursue the Chernoff technique for the hard-limiting correlation receiver.

To understand the application of the Chernoff bound, we first write

$$\overline{P}_e(\underline{\tau}) = E_{\underline{\phi},\underline{b}}\left\{\Pr\left[Y_N \geq 1 \mid b_0^{(1)} = -1, \rho_I\right]\right\}. \qquad (7.9)$$

Then, we define a new zero mean random variable as

$$\hat{\eta}_j \triangleq \hat{Z}_j^{(1)} - E\{\hat{Z}_j^{(1)} \mid b_0^{(1)} = -1, \rho_I\}, \qquad (7.10)$$

where $\hat{Z}_j^{(1)}$ is given in (7.2). Now we can rewrite (7.9) as

$$\overline{P}_e(\underline{\tau}) = E_{\underline{\phi},\underline{b}}\left\{\Pr\left[\sum_{j=0}^{N-1}\hat{\eta}_j \geq 1 - \sum_{j=0}^{N-1}\mu_j \mid b_0^{(1)} = -1, \rho_I\right]\right\}, \qquad (7.11)$$

where

$$\mu_j \triangleq E\left\{\hat{Z}_j^{(1)} \mid b_0^{(1)} = -1, \rho_I\right\} = 2\Psi\left[\frac{1 - I_j^{(1)}}{\sigma}\right] - 1. \qquad (7.12)$$

In (7.12) $\Psi(a)$ is defined as $\Pr[\overline{\eta} > a]$, where $\overline{\eta}$ is a typical normalized noise sample with unit variance. Now if we define μ_s as $\sum_{j=0}^{N-1}\mu_j$ and then we can write

$$\bar{P}_e(\underline{\tau}) = E_{\underline{\phi},\underline{b}} \left\{ \sum_{m=1}^{\frac{N+1}{2}} \Pr \left[\hat{\eta}_s = 2m - 1 - \mu_s \mid b_0^{(1)} = -1, \rho_I \right] \right\}, \tag{7.13}$$

where $\hat{\eta}_s \overset{\Delta}{=} \sum_{j=0}^{N-1} \hat{\eta}_j$. Now, for each $t > 0$, we introduce a discrete random variable ζ_t with mass function

$$\Pr[\zeta_t = a] = \frac{e^{ta} \Pr[\hat{\eta}_s = a]}{M_{\hat{\eta}_s}(t)}, \quad t>0 \tag{7.14}$$

where $M_{\hat{\eta}_s}$ is the moment generating function of the random variable $\hat{\eta}_s$. Replacing $\hat{\eta}_s$ with ζ_t, the expression inside the curly bracket in (7.13) can be bounded from above as

$$\sum_{m=1}^{\frac{N+1}{2}} \Pr \left[\hat{\eta}_s = 2m - 1 - \mu_s \right] = M_{\hat{\eta}_s}(t) \sum_{m=1}^{\frac{N+1}{2}} \Pr \left[\zeta_t = 2m - 1 - \mu_s \right] e^{t(\mu_s + 1 - 2m)}$$

$$\leq \exp\left[m_{\hat{\eta}_s}(t) - t(1 - \mu_s) \right] \sum_{m=1}^{\frac{N+1}{2}} \Pr \left[\zeta_t = 2m - 1 - \mu_s \right], \tag{7.15}$$

where $m_{\hat{\eta}_s}(t) \overset{\Delta}{=} \ln[M_{\hat{\eta}_s}(t)]$. Finally, for any value of t on the positive real line we have

$$\bar{P}_e(\underline{\tau}) \leq E_{\underline{b},\underline{\phi}} \left\{ \exp \left[t\mu_s - t + m_{\hat{\eta}_s}(t) \right] \right\}. \tag{7.16}$$

To get the best bound we minimize the right-hand side of (7.16) with respect to t. In particular, we are looking for t_0 such that

$$t_0 = \arg\left\{\min_{t>0}\left[m_{\hat{\eta}_o} + t\,\mu_s - t\right]\right\}. \tag{7.17}$$

Differentiating the argument in (7.17) and setting the result equal to zero results in

$$m'_{\hat{\eta}_o}(t) = 1 - \mu_s \;. \tag{7.18}$$

In order to see whether a solution to (7.17) exists and it satisfies (7.18), we write the expression for $m_{\hat{\eta}_o}(t)$ for the hard-limiting correlation receiver as

$$m_{\hat{\eta}_o}(t) = \sum_{j=0}^{N-1} \ln\left[e^{(1-\mu_j)t}\left(\frac{1+\mu_j}{2}\right) + e^{-(1+\mu_j)t}\left(\frac{1-\mu_j}{2}\right)\right]. \tag{7.19}$$

Using (7.19), it is easy to show that t_0 satisfying (7.18) is obtained by solving

$$\sum_{j=0}^{N-1} \frac{1-\mu_j-(1+\mu_j)\,e^{2t_0}}{1-\mu_j+(1+\mu_j)\,e^{2t_0}} = 0 \;,\; t_0>0 \;. \tag{7.20}$$

From (7.19) it follows that a solution to (7.20) exists and satisfies (7.17). Finally, the Chernoff bound on the error probability of the hard-limiting correlation receiver is given as

$$\bar{P}_e(\underline{\tau}) \le E_{\underline{b},\underline{\phi}}\left\{\exp\left[t_0\,\mu_s - t_0 + m_{\hat{\eta}_o}(t_0)\right]\right\}, \tag{7.21}$$

where t_0 is obtained by solving (7.20). We examine the tightness of the Chernoff upper bound on $\overline{P}_e(\underline{\tau})$ for the ε-mixture and Laplacian examples. Typical results accessing the accuracy of this approximation are depicted in Tables 7.3 and 7.4. These results are obtained for the same DS/SSMA system considered in the examples in Chapter 5. These examples are carried out with lengths of signature sequences equal to 31, 63, 127. Tables 7.3 and 7.4 correspond to SNR = 8 and SNR = 4, respectively. As expected, this bound is not particularly tight; however, its simplicity and the lack of alternative bounds makes it potentially useful.

7.4. Asymptotic Analysis

In Section 7.2 an approximation was obtained for the bit-error probability of hard-limiting correlation receivers in impulsive noise that was good for large N. Also in Chapter 5, more significant improvement in performance of the hard-limiting correlator over the linear correlator was experienced as we used longer signature sequence lengths. Motivated by these findings, here we investigate the behavior of hard-limiting DS/SSMA correlation receivers when the length of the signature sequences used per data bit becomes infinitely large.

Recall from the asymptotic analysis in Section 6.2 that conditioned on the multiple-access parameters ρ_I, the test statistic is a sum of N independent random variables. Therefore, assuming that

$$\lim_{N \to +\infty} \frac{E\{Y_N \mid b_0^{(1)}=+1, \rho_I\}}{\sqrt{Var(Y_N \mid b_0^{(1)}=+1, \rho_I)}} \quad \text{exists almost surely}, \tag{7.22}$$

the average bit-error probability can be written asymptotically as

$$\overline{P}_e = E_{\underline{\tau}, \underline{\phi}, \underline{b}} \left\{ Q\left[\frac{E_Y}{\sigma_Y}\right] \right\}, \tag{7.23}$$

TABLE 7.3. CHERNOFF UPPER BOUND FOR THE ERROR PROBABILITY OF HARD-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND IMPULSIVE CHANNELS, SNR=8.0 dB, K=2 AND TYPICAL TIME DELAYS.

DISTRIBUTION	N	P_e	P_e^U
GAUSSIAN	31	4.63×10^{-3}	2.43×10^{-2}
	63	2.51×10^{-3}	1.38×10^{-2}
	127	2.77×10^{-3}	1.50×10^{-2}
LAPLACIAN	31	8.67×10^{-4}	5.22×10^{-3}
	63	1.02×10^{-4}	6.28×10^{-4}
	127	5.41×10^{-5}	2.63×10^{-4}
ε-MIXTURE $\varepsilon=0.01, \gamma^2=100$	31	3.90×10^{-4}	2.67×10^{-3}
	63	5.77×10^{-5}	3.61×10^{-4}
$\varepsilon=0.1, \gamma^2=100$	31	2.59×10^{-6}	2.13×10^{-5}
	63	2.06×10^{-10}	1.17×10^{-9}

TABLE 7.4. CHERNOFF UPPER BOUND FOR THE ERROR PROBABILITY OF HARD-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GAUSSIAN AND IMPULSIVE CHANNELS, SNR=4.0 dB, K=2 AND TYPICAL TIME DELAYS.

DISTRIBUTION	N	P_e	P_e^U
GAUSSIAN	31	4.14×10^{-2}	1.65×10^{-1}
	63	3.68×10^{-2}	1.62×10^{-1}
LAPLACIAN	31	1.01×10^{-2}	4.75×10^{-2}
	63	4.84×10^{-3}	2.56×10^{-2}
ε-MIXTURE ε=0.01, γ^2=100	31	9.72×10^{-3}	4.66×10^{-2}
	63	6.54×10^{-3}	3.39×10^{-2}
ε=0.1, γ^2=100	31	6.66×10^{-5}	5.18×10^{-4}
	63	7.61×10^{-7}	4.84×10^{-6}

where $E_Y \triangleq \lim_{N\to+\infty} E\left\{\frac{Y_N}{\sqrt{N}} \mid b_0^{(1)}=+1, \rho_I\right\}$ and $(\sigma_Y)^2 \triangleq \lim_{N\to+\infty} \mathrm{Var}\left[\frac{Y_N}{\sqrt{N}} \mid b_0^{(1)}=+1, \rho_I\right]$. Note that here Y_N is given in (7.2). For a given set of multiple-access parameters the normalized mean can be written as

$$E_Y = \lim_{N\to+\infty} \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \left\{ 1-2\Psi\left[\frac{\alpha}{\sqrt{N}}\,[\,1+I_j^{(1)}\,]\right]\right\}, \tag{7.24}$$

where $\Psi(a) \triangleq \Pr[\,\bar{\eta} > a\,]$ is the complementary distribution function of a typical noise sample $\bar{\eta}$, normalized to have zero mean and unit variance. For the ε-mixture example, the distribution function in (7.24) is given as

$$\Psi(a) = (1-\varepsilon)\,Q\left[a\sqrt{1-\varepsilon+\varepsilon\gamma^2}\right] + \varepsilon\,Q\left[a\sqrt{\varepsilon+\frac{1-\varepsilon}{\gamma^2}}\right]. \tag{7.25}$$

Similarly for the Laplacian example we have

$$\Psi(a) = \frac{1}{2}\exp\{-\sqrt{2}a\}, \quad \text{for } a \geq 0. \tag{7.26}$$

Referring to (7.24), the probability distribution function can be expanded in Taylor series about $\frac{\alpha}{\sqrt{N}}$ as

$$\Psi\left[\frac{\alpha}{\sqrt{N}}[1+I_j^{(1)}]\right] = \Psi\left(\frac{\alpha}{\sqrt{N}}\right) + \sum_{n=1}^{\infty} \frac{\alpha^n (I_j^{(1)})^n}{N^{\frac{n}{2}} n!}\,\Psi^{(n)}\left(\frac{\alpha}{\sqrt{N}}\right), \tag{7.27}$$

where $\Psi^{(n)}$ is the n^{th} derivative of Ψ. Now if $\Psi(.)$ is analytic and we have the following asymptotic property:

$$\lim_{N\to+\infty} \sum_{j=0}^{N-1} \sum_{n=1}^{\infty} \frac{\alpha^n (I_j^{(1)})^n}{N^{\frac{n+1}{2}} n!} \Psi^{(n)}\left(\frac{\alpha}{\sqrt{N}}\right) = 0 \quad (a.s.), \tag{7.28}$$

then we can write

$$E_{\bar{Y}} = \lim_{N\to+\infty} \sqrt{N} \left[1 - 2\Psi\left(\frac{\alpha}{\sqrt{N}}\right) \right] = 2\alpha f_{\bar{\eta}}(0) , \tag{7.29}$$

where $f_{\bar{\eta}}$ is the probability density function of $\bar{\eta}$. From (7.2) and (7.29), it follows that $(\sigma_{\bar{Y}})^2$ is equal to 1. This yields a simple expression for the asymptotic error probability of the hard-limiting correlation receiver as

$$\lim_{N\to+\infty} \bar{P}_e = Q\left\{ 2\alpha f_{\bar{\eta}}(0) \right\}, \tag{7.30}$$

if (7.28) is satisfied. Note that $Q(2\alpha f_{\bar{\eta}}(0))$ is the asymptotic single-user performance of the hard-limiting correlator (see Chapter 3). Thus, (7.30) implies that single-user asymptotic performance is achieved by the hard-limiting correlation receiver if the condition (7.28) is satisfied.

Regarding condition (7.28), note that the order of summations can be interchanged if

$$\lim_{N\to+\infty} \sum_{j=0}^{N-1} \sum_{n=1}^{\infty} \left| \frac{\alpha^n (I_j^{(1)})^n}{N^{\frac{n+1}{2}} n!} \Psi^{(n)}\left(\frac{\alpha}{\sqrt{N}}\right) \right| < +\infty \quad (a.s.) . \tag{7.31}$$

Assuming that $|I_j^{(1)}| \leq K$ and that there exists $u \in R$ such that $\left| \Psi^{(n)}\left(\frac{\alpha}{\sqrt{N}}\right) \right| \leq u$ for all n and large enough N then condition (7.31) is satisfied if the number of users is finite. However, (7.31) can hold for infinitely large K, for instance, it is easy to see that (7.31) will be satisfied if

$$\sum_{j=0}^{N-1} \frac{|I_j|^n}{N^{\frac{n+1}{2}}} < +\infty \quad (\text{a.s.}) \;\forall n \; . \tag{7.32}$$

By induction we can show that the conditions

$$\lim_{N\to+\infty} I_1(\underline{b},\underline{\tau},\underline{\phi}) = 0 \text{ and } \lim_{N\to+\infty} \sum_{j=0}^{N-1} \frac{|I_j^{(1)}|^2}{N^{3/2}} = 0 \quad (\text{a.s.}) \; , \tag{7.33}$$

are sufficient to meet (7.28) and (7.31) if $K \le N^{1/4}$. Note that the first part of (7.33) is identical to the condition for the linear correlator (6.21). The second part is satisfied by imposing slightly more restrictive upper bound on the number of users. Thus, the ability to achieve a single-user performance asymptotically in N is possible within the above regularity conditions, and so Table 3.1 and Figures 3.8 and 3.9 contain the asymptotic average bit-error probability for the hard-limiting correlation receiver within these conditions.

CHAPTER 8

NONLINEAR CORRELATION RECEIVERS IN IMPULSIVE CHANNELS: ASYMPTOTIC ANALYSIS

In Chapters 6 and 7, we investigated the asymptotic performance of linear and hard-limiting DS/SSMA correlation receivers as the lengths of the signature sequences grew without bound. In turn, asymptotically ideal DS/SSMA signals were proposed. To gain further insight into the non-linear problem and explore the fundamental limitations of these systems, we carried out an asymptotic analysis for general nonlinear DS/SSMA correlation receivers depicted in Figure 8.1, where g is a general memoryless, nonlinear element. Then, we searched for necessary asymptotic properties for the spreading sequences that assure asymptotic single-user performance in a multi-user environment.

Recall that in the asymptotic analysis the energy per data bit for a user, the length of the data period T, the noise power, and in turn the signal-to-noise ratio are kept constant. However, the length of the signature sequences used per data bit is growing without bound, and in turn the sampling (and chip) period T_c is decreasing to zero. Also note that we rescaled the data by $\sqrt{N}$ to keep the correct scale (this prevents us from having to allow the nonlinearity to change with N), and therefore the test statistic is written as

$$Y_N = \sum_{j=0}^{N-1} g\left[\tilde{\eta}_j + \sqrt{N}\, I'^{(1)}_j + \sqrt{\frac{P_1}{2N}}\, T\, b_0^{(1)}\right], \tag{8.1}$$

where $\tilde{\eta}_j$'s are samples of noise with zero mean and variance $\frac{N_0 T}{4}$ and g is the nonlinear element. In (8.1) the $I'^{(1)}_j$'s denote the contribution of multiple-access interference in the samples of the received signal and are defined as

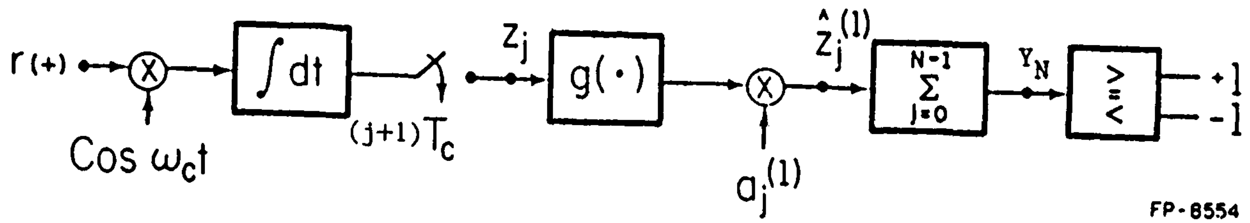

Figure 8.1. Structure of a DS/SSMA nonlinear correlation receiver.

$$I_j'^{(1)} \triangleq \sqrt{\frac{P_1}{2}}\, T_c I_j^{(1)} \qquad j=0,1,\ldots,N-1, \tag{8.2}$$

where $I_j^{(1)}$ is given in (5.12). Note that $I_j'^{(1)}$ relates to $I_1(\underline{b},\underline{\tau},\underline{\phi})$ defined earlier in (4.5) through

$$I_1(\underline{b},\underline{\tau},\underline{\phi}) = \frac{\sum_{j=0}^{N-1} I_j'^{(1)}}{\sqrt{\frac{P_1}{2}}\, T}. \tag{8.3}$$

Referring to (8.1), conditioned on the multiple-access parameters ρ_I, Y_N is seen to be the sum of N independent random variables. To apply the central limit theorem to the sum (8.1), we first assume that

$$\lim_{N\to+\infty} \frac{E\{\, Y_N \mid b_0^{(1)}=+1, \rho_I \,\}}{\sqrt{\mathrm{Var}(\, Y_N \mid b_0^{(1)}=+1, \rho_I \,)}} \quad \text{exists almost surely}, \tag{8.4}$$

then, via the bounded convergence theorem, the average bit-error probability for the nonlinear DS/SSMA receiver can be written asymptotically as

$$\bar{P}_e = E_{\underline{\tau},\underline{\phi},\underline{b}} \left\{ Q\left[\frac{E_Y}{\sigma_Y} \right] \right\}, \tag{8.5}$$

where $E_Y \triangleq \lim_{N\to+\infty} E\left\{ \frac{Y_N}{\sqrt{N}} \mid b_0^{(1)}=+1, \rho_I \right\}$ and $(\sigma_Y)^2 \triangleq \lim_{N\to+\infty} \mathrm{Var}\left[\frac{Y_N}{\sqrt{N}} \mid b_0^{(1)}=+1, \rho_I \right]$. For a given set of multiple-access parameters the normalized conditional mean can be written as

$$E\gamma = \lim_{N\to+\infty} \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \int_0^{\infty} \left[g(x+\theta_N[1+I_j^{(1)}]) - g(x-\theta_N[1+I_j^{(1)}]) \right] f_{\tilde{\eta}_j}(x)\, dx, \tag{8.6}$$

where $\theta_N \triangleq \sqrt{\frac{P_1}{2N}}\,T$. Similarly, the variance of the normalized test statistic has the form

$$(\sigma_\gamma)^2 = \lim_{N\to+\infty} \frac{1}{N} \sum_{j=0}^{N-1} \int_0^{\infty} \left[g^2(x+\theta_N[1+I_j^{(1)}]) + g^2(x-\theta_N[1+I_j^{(1)}]) \right] f_{\tilde{\eta}_j}(x)\, dx$$

$$- \lim_{N\to+\infty} \frac{1}{N} \sum_{j=0}^{N-1} \left\{ \int_0^{\infty} \left[g(x+\theta_N[1+I_j^{(1)}]) - g(x-\theta_N[1+I_j^{(1)}]) \right] f_{\tilde{\eta}_j}(x)\, dx \right\}^2. \tag{8.7}$$

In (8.6) and (8.7) if conditions of the dominated convergence theorem are met by the integrands, then the order of the infinite sums and the integrals can be interchanged (see [6] page 257). We first concentrate on (8.6); if $g(\cdot)$ is analytic and has a bounded second derivative then we expand the integrands in (8.6) as

$$g(x+\theta_N[1+I_j^{(1)}]) = g(x+\theta_N) + \theta_N I_j^{(1)} g'(x+\theta_N) + \frac{\theta_N^2 \,|\, I_j^{(1)} |^2}{2} g''(\xi), \tag{8.8}$$

for some $\xi \in (x+\theta_N, x+\theta_N[1+I_j^{(1)}])$. Substituting (8.8) into (8.6), we realize that if we have the following asymptotic properties

$$\frac{(\alpha\beta)^2}{2} \sup(g'') \lim_{N\to+\infty} \frac{1}{N^{\frac{3}{2}}} \sum_{j=0}^{N-1} |\, I_j^{(1)} |^2 = 0 \quad (\text{a.s.}), \tag{8.9}$$

and

$$\alpha\beta \lim_{N\to+\infty} \sum_{j=0}^{N-1} \frac{I_j^{(1)}}{N} \int_0^{+\infty} \left[g'(x+\theta_N)+g'(x-\theta_N)\right] f_{\tilde{\eta}}(x)\,dx = 0 \quad (a.s.), \tag{8.10}$$

with $\alpha \triangleq \sqrt{\frac{2E_b^{(1)}}{N_0}}$, and $\beta \triangleq \sqrt{\frac{N_0 T}{4}}$, then we can simplify (8.6) as

$$E_{\tilde{Y}} = \int_0^{+\infty} \lim_{N\to+\infty} \sqrt{N} \left[g(x+\theta_N)-g(x-\theta_N)\right] f_{\tilde{\eta}}(x)\,dx, \tag{8.11}$$

where $f_{\tilde{\eta}}(.)$ is the probability density function of a typical zero mean noise sample which has variance β^2. Note that (8.11) can be simplified further to

$$E_{\tilde{Y}} = \int_0^{\infty} \alpha\beta\, g'(x) f_{\tilde{\eta}}(x)\,dx, \tag{8.12}$$

Similarly for $(\sigma_{\tilde{Y}})^2$ in (8.7), since g^2 is analytic and assuming that g^2 has bounded second derivative, then the integrands can be expanded in a Taylor series about a point near x, i.e.,

$$g^2(x+\theta_N[1+I_j^{(1)}]) = g^2(x+\theta_N) + \theta_N I_j^{(1)} (g^2)'(x+\theta_N) + \frac{\theta_N^2 \,|\, I_j^{(1)} \,|^2}{2} (g^2)''(\xi), \tag{8.13}$$

for some $\xi \in \left[x+\theta_N, x+\theta_N[1+I_j^{(1)}]\right]$. In (8.13) $(g^2)''$ is the 2^{nd} derivative of g^2. Substituting (8.13) into (8.7), we realize that if we have the following asymptotic properties

$$\frac{(\alpha\beta)^2}{2} \sup((g^2)'') \lim_{N\to+\infty} \frac{1}{N^2} \sum_{j=0}^{N-1} |\, I_j^{(1)} \,|^2 = 0 \quad (a.s.), \tag{8.14}$$

$$\alpha\beta \lim_{N\to+\infty} \frac{1}{N^{\frac{3}{2}}} \sum_{j=0}^{N-1} I_j^{(1)} \int_0^{+\infty} \left[(g^2)'(x+\theta_N) - (g^2)'(x-\theta_N) \right] f_{\tilde{\eta}}(x)\, dx = 0 \quad (\text{a.s.}), \tag{8.15}$$

and also

$$\left| \int_0^{+\infty} \left[g(x+\theta_N[1+I_j^{(1)}]) - g(x-\theta_N[1+I_j^{(1)}]) \right] f_{\tilde{\eta}}(x)\, dx \right| < \frac{1}{N^d} \quad (\text{a.s.}) \ , \tag{8.16}$$

for some $d > 0$ and all $j = 0, 1, \ldots, N - 1$, then we can simplify (8.9) as

$$(\sigma_{\tilde{Y}})^2 = 2\int_0^{\infty} g^2(x)\, f_{\tilde{\eta}}(x)\, dx. \tag{8.17}$$

Finally, (8.12) and (8.17) yield the following expression for the argument of the Q function in (8.5):

$$\frac{E_{\tilde{Y}}}{\sigma_{\tilde{Y}}} = \frac{2\alpha\beta \int_0^{\infty} g'(x)\, f_{\tilde{\eta}}(x)\, dx}{\left[2\int_0^{\infty} g^2(x)\, f_{\tilde{\eta}}(x)\, dx \right]^{½}}, \tag{8.18}$$

if the conditions (8.9), (8.10), (8.14), (8.15), and (8.16) are satisfied. By substituting (8.18) into the expression for the asymptotic error probability given in (8.5), we recognize the possibility of single-user performance as

$$\lim_{N\to+\infty} \overline{P}_e = Q\left[T\sqrt{\frac{P_1}{2}\, v} \right], \tag{8.19}$$

where ν is the detection efficacy defined in (3.11), which for the nonlinear correlation receiver is easily derived as

$$\nu = \frac{2\left[\int_0^{\infty} g'(x)\, f_{\bar{\eta}}(x)\, dx\right]^2}{\int_0^{\infty} g^2(x)\, f_{\bar{\eta}}(x)\, dx}. \tag{8.20}$$

We have just shown that asymptotically in N and within regularity on the nonlinearity the nonlinear correlation receiver can perform with single-user error probability in a multi-user environment if the conditions (8.9), (8.10), (8.14), (8.15), and (8.16) are satisfied. Since we assumed that both g and g^2 have bounded second derivative, the above conditions are satisfied if

$$\lim_{N \to +\infty} I_1(\underline{b}, \underline{\tau}, \underline{\phi}) = 0 \text{ and } \lim_{N \to +\infty} \sum_{j=0}^{N-1} \frac{|I_j^{(1)}|^2}{N^{3/2}} = 0 \quad (a.s.), \tag{8.21}$$

if the number of users does not grow faster than $N^{1/4}$. Note that the first part of (8.21) is identical to the condition for the linear correlator (6.21). The second part is satisfied by imposing slightly more restrictive upper bounds on the number of users, and is the same as the analogous condition derived for the hard-limiting correlator.

To study further the condition (8.21), we first recall from (6.2) that the multiple-access interference can be written as

$$I_1(\underline{b}, \underline{\tau}, \underline{\phi}) = \sum_{k=2}^{K} \sqrt{\epsilon_{k,1}} \cos\phi_k \frac{W_{k,1}}{N}, \tag{8.22}$$

where

$$W_{k,1} = b_{-1}^{(k)} [\, d_k C_{k,1}(m_k - N + 1) + (1-d_k) C_{k,1}(m_k - N) \,]$$

$$+ b_0^{(k)} [\, d_k C_{k,1}(m_k+1) + (1-d_k) C_{k,1}(m_k) \,]. \tag{8.23}$$

In (8.23) $d_k \triangleq \frac{N\tau_k}{T} - \left\lfloor \frac{N\tau_k}{T} \right\rfloor$, $m_k \triangleq \left\lfloor \frac{N\tau_k}{T} \right\rfloor$, and $C_{k,1}$ is the discrete aperiodic cross-correlation function defined in (4.10). For a finite number of users and from (8.22) and (8.23) it follows that (8.21) is satisfied if we have the following:

$$\lim_{N \to +\infty} \frac{1}{N} \max_m \left[C_{k,i}(m) \right] = 0, \tag{8.24}$$

for all k,i such that $k \neq i$. This is equivalent to the conditions

$$\lim_{N \to +\infty} \max_m \frac{1}{N} \mid \theta_{k,i}(m) \mid = 0 \quad \text{and} \quad \lim_{N \to +\infty} \max_m \frac{1}{N} \mid \hat{\theta}_{k,i}(m) \mid = 0, \tag{8.25}$$

for all k and i such that $k \neq i$. As mentioned earlier, the existence of binary sequences satisfying these conditions follows by applying (8.24) to infinite sequences proposed in [34]. For these sequences the convergence of the crosscorrelations to zero is very slow. However, for the infinite-user case the condition (8.22) is still satisfied as long as the number users do not grow faster than $N^{\frac{1}{4}}$.

An important issue to discuss here is the interpretation of the desired condition (8.25) in terms of bandwidth requirements. First we write the discrete Fourier transform of the signature sequence $\underline{a}^{(k)}$ as

$$A_m^{(k)} \triangleq \frac{1}{N} \sum_{j=0}^{N-1} a_j^{(k)} W_N^{jm} \quad m=0,1,\ldots,N-1, \tag{8.26}$$

where $W_N \stackrel{\Delta}{=} \exp\{-i\frac{2\pi}{N}\}$. Let $\Theta_{k,i}(\cdot)$ denote the DFT of the periodic crosscorrelation function $\theta_{k,i}(\cdot)$. It is well known that

$$\Theta_{k,i}(m) = N\, A_{-m}^{(k)} A_m^{(i)} \quad m=0,1,\ldots,N-1. \tag{8.27}$$

From the discussion in [34], a large set of infinitely long sequences can be generated such that for these sequences (8.25) approaches zero as fast as N^{-u} for some $0<u<\frac{1}{2}$. For the sequences in the set S_N, $\Theta_{k,i}(m)$ grows with N, but as slow as N^{1-u} with $0<u<\frac{1}{2}$ for all m. From (8.27) it follows that $A_{-m}^{(k)} A_m^{(i)}$ approaches zero as fast as N^{-u} for all m and all pairs of sequences in S_N. In summary, (8.25) implies that

$$\lim_{N \to +\infty} A_{-m}^{(k)} A_m^{(i)} = 0 \quad \text{for all } m \in \mathbf{Z}. \tag{8.28}$$

This indicates that the signals are occupying different frequency bands which causes the infinite-user communication system with asymptotically ideal sequences to use infinitely large bandwidth.

To illustrate the frequency domain characteristics of these sequences consider the following example. Suppose the signature sequence $\underline{a}^{(1)}$ with $a_j^{(1)} = +1$ for all $0 \le j \le N-1$ is a member of the set of asymptotically ideal sequences. Then the DFT of $\underline{a}^{(1)}$ is

$$A_m^{(1)} = \begin{cases} 1 & \text{if } m=0 \\ 0 & \text{if } m=1,\ 2,\ldots,N-1 \end{cases} \tag{8.29}$$

Now as N grows without bound, the condition (8.28) implies that all other signature sequences in S_N must have

$$\lim_{N \to +\infty} A_0^{(k)} = 0 \qquad \text{when } k \neq 1. \tag{8.30}$$

Hypothetically, the set of K sequences can be built by forcing the DFT of $\underline{a}^{(k)}$ to have a nonzero value only at one point, $A_{k-1}^{(k)} \neq 0$. Thus (8.28) is satisfied; however the spectrum is completely occupied by the infinitely many users. This is not surprising, since single-user performance is achieved in an infinite-user environment.

In this chapter we have analyzed the performance of nonlinear DS/SSMA correlation receivers as we let N grow without bound. Assuming some regularities on the nonlinearity, conditions for multiple-access signals were obtained to have asymptotic single-user error probabilities in an infinite-user environment. This limiting behavior of $\overline{P}_e$ implies that the performance of multi-user communication systems in impulsive channels can , for long sequences, be improved significantly, knowing the statistical characteristic of the channel noise.

CHAPTER 9

SOFT-LIMITING CORRELATION RECEIVERS IN IMPULSIVE CHANNELS: APPROXIMATIONS

The results of Chapter 5 indicated that the improvement in performance by using the hard-limiting receiver in place of the linear correlation receivers increases as the tails of the noise distribution become heavier (recall that the SNR was fixed). However, for less impulsive noise examples (e.g., Laplacian noise), we observed that for small N and K=2 the conventional linear receiver outperforms the hard-limiting correlation receiver. Comparing the two receivers, the linear correlator is more effective against multiple-access interference, whereas the hard-limiting correlation receiver is more effective against impulsive channel noise. We combined these desirable features and introduced soft-limiting correlation receivers as an alternative for moderately impulsive channels. The performance of the soft-limiting correlator will be investigated in this chapter.

The soft limiter is the optimum nonlinearity for detecting the presence of a class of signals in Laplacian noise. In this case the memoryless nonlinearity is the clipper function defined as

$$\mathrm{cl}(x) = \begin{cases} -c & x < -c \\ x & -c \le x < c \\ c & c \le x \end{cases}, \tag{9.1}$$

where c is the clipping level. Although it is a suboptimum nonlinearity in a multi-user environment, unlike the hard-limiter, the linear portion of the soft-limiter allows passage of the multiple-access signal. Similar to the hard-limiter, the limiting part of the soft-limiter controls the effects of non-Gaussian impulsive noise samples (see Figure 9.1).

Computing the average bit-error probability of the soft-limiting correlation receiver in a multi-user system is extremely complicated. Therefore, in this thesis, we resort to asymptotic performance analysis and an upper bound on the multi-user error probability.

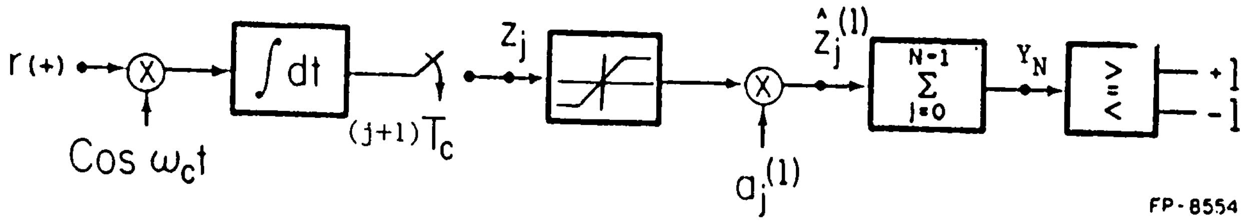

Figure 9.1. Structure of a DS/SSMA soft-limiting correlation receiver.

9.1. Chernoff Upper Bound

In this section, we bound the error probability of the soft-limiting correlation receiver in a multi-user environment by applying the Chernoff bound. We begin by writing the error probability as

$$\overline{P}_e(\underline{\tau}) = E_{\underline{\phi},\underline{b}}\left\{\Pr\left[Y_N = 0 \mid b_0^{(1)} = -1, \rho_I\right]\right\}, \tag{9.2}$$

where the test statistic Y_N is given as

$$Y_N = \sum_{j=0}^{N-1} \hat{Z}_j^{(1)} = \sum_{j=0}^{N-1} \mathrm{cl}\left[\eta'_j + I'^{(1)}_j + \sqrt{\frac{P_1}{2}} T_c b_0^{(1)}\right]. \tag{9.3}$$

cl is the clipper function. Now, we define a zero mean random variable by

$$\hat{\eta}_j \triangleq \hat{Z}_j^{(1)} - E\{\hat{Z}_j^{(1)} \mid b_0^{(1)} = -1, \rho_I\}, \tag{9.4}$$

where $\hat{Z}_j^{(1)}$ is given in (9.3). By substituting (9.4) into (9.2) we can rewrite the error probability as

$$\overline{P}_e(\underline{\tau}) = E_{\underline{\phi},\underline{b}}\left\{\Pr\left[\sum_{j=0}^{N-1} \hat{\eta}_j \geq -\sum_{j=0}^{N-1} E\{\hat{Z}_j^{(1)}\} \mid b_0^{(1)} = -1, \rho_I\right]\right\}. \tag{9.5}$$

Note that in (9.5) the mean of $\hat{Z}_j^{(1)}$ is the conditional expectation of $\hat{Z}_j^{(1)}$ given $b_0^{(1)} = -1$ and is given as

$$E\{\hat{Z}_j^{(1)} \mid b_0^{(1)}=-1, \rho_I\} = c\Pr\left[\eta_j^{(1)} \ge c'+1-I_j^{(1)}\right] - c\Pr\left[\eta_j^{(1)} < -c'+1-I_j^{(1)}\right]$$

$$+ \int_{-c'+1-I_j^{(1)}}^{c'+1-I_j^{(1)}} \delta\left[x+I_j^{(1)}-1\right] f_{\eta_j^{(1)}}(x)\,dx, \tag{9.6}$$

where $\delta \triangleq \sqrt{\frac{P_1}{2}}T_c$ and $c' \triangleq \frac{c}{\delta}$. In (9.6), $f_{\eta_j^{(1)}}$ denotes the probability density function of a typical noise sample $\eta_j^{(1)}$ which has zero mean and variance $\frac{N_0 N}{2E_b^{(1)}}$. Similar to the analysis in Chapter 7, the expression for the error probability can be bounded from above as

$$\overline{P}_e(\underline{\tau}) \le E_{\underline{\phi},\underline{b}}\left\{\exp\left[t\mu_s + m_{\eta_s}(t)\right]\right\} \quad t>0, \tag{9.7}$$

where $\mu_s \triangleq \sum_{j=0}^{N-1} E\{\hat{Z}_j^{(1)} \mid b_0^{(1)}=-1, \rho_I\}$, and $m_{\eta_s}(t) \triangleq \ln[M_{\eta_s}(t)]$. Here $M_{\eta_s}(t)$ is the moment generating function of the random variable $\eta_s \triangleq \sum_{j=0}^{N-1}\eta_j$ conditioned on $b_0^{(1)}=-1$ and ρ_I. Note that, conditioned on the multiple-access interference, the η_j's are independent, but not identically distributed. Therefore, $M_{\eta_s}(t)$ can be expressed as the product of the moment generating functions of the η_j's which are given as

$$M_{\eta_j}(t) = e^{(c-\mu_j)t}\Pr\left[\eta_j^{(1)} \ge c'+1-I_j^{(1)}\right] + e^{(-c-\mu_j)t}\Pr\left[\eta_j^{(1)} < -c'+1-I_j^{(1)}\right]$$

$$+ \int_{-c'+1-I_j^{(1)}}^{c'+1-I_j^{(1)}} \exp\left[t[\delta(x+I_j^{(1)}-1)-\mu_j]\right] f_{\eta_j^{(1)}}(x)\,dx, \tag{9.8}$$

where $\mu_j \triangleq E\{\hat{Z}_j \mid b_0^{(1)}=-1, \rho_I\}$. Recall that (9.7) holds for any positive value of t; therefore, to achieve the best bound we minimize the right-hand side of (9.7) with respect to t. In particular, we are looking for t_0 such that

$$t_0 = \arg\left\{\min_{t>0}\left[m_{\eta_s}(t) + t\mu_s\right]\right\}. \tag{9.9}$$

Similar to the derivation in Chapter 7, differentiating the argument in (9.9) and setting the result equal to zero yields the following:

$$\sum_{j=0}^{N-1} \frac{M'_{\eta_j}(t_0)}{M_{\eta_j}(t_0)} + \mu_j = 0 \quad t_0>0. \tag{9.10}$$

If (9.10) can be solved, the Chernoff bound on the error probability of the soft-limiting correlation receiver is given as

$$\overline{P}_e(\underline{\tau}) \le E_{\underline{\varphi},\underline{b}}\left\{\exp\left[t_0\mu_s + m_{\eta_s}(t_0)\right]\right\}, \tag{9.11}$$

where t_0 is obtained by solving (9.10).

Due to the complexity of the expression for $M_{\eta_s}(t)$, we pursue this technique only for the Laplacian channel example. The Laplacian density function is given by

$$f_{\eta_j^{(1)}}(x) = \frac{1}{\sqrt{2}\sigma} e^{-\frac{\sqrt{2}}{\sigma}|x|}, \tag{9.12}$$

where the variance of $\eta_j^{(1)}$ is $\sigma^2 = \dfrac{N_0 N}{2E_b^{(1)}}$. The cumulative distribution function for this example is given as

$$F_{\eta_j^{(1)}}(a) = 1 - \frac{1}{2} e^{-\frac{\sqrt{2}}{\sigma} a} \quad a \geq 0. \tag{9.13}$$

Considering (9.12) and (9.13), the expression for the conditional mean of $\hat{Z}_j$ can be simplified as

$$E\{\hat{Z}_j \mid b_0^{(1)} = -1, \rho_I\} = -\delta \left[1 - I_j^{(1)} - \frac{\sigma}{\sqrt{2}} e^{-\frac{\sqrt{2}}{\sigma} c'} \sinh[\frac{\sqrt{2}}{\sigma} (1 - I_j^{(1)})] \right]. \tag{9.14}$$

Similarly, the moment-generating function of $\hat{\eta}_j$ given in (9.8) can be simplified to

$$M_{\hat{\eta}_j}(t) = \frac{1}{2} e^{-\mu_j t - \frac{\sqrt{2}}{\sigma} c'} \left[\frac{e^{tc - \frac{\sqrt{2}}{\sigma}[1 - I_j^{(1)}]}}{1 - \frac{\sqrt{2}}{t\delta\sigma}} + \frac{e^{-tc + \frac{\sqrt{2}}{\sigma}[1 - I_j^{(1)}]}}{1 + \frac{\sqrt{2}}{t\delta\sigma}} \right] - \frac{2}{t^2\delta^2\sigma^2 - 2} e^{-t[\delta(1 - I_j^{(1)}) + \mu_j]}, \tag{9.15}$$

where $\delta \triangleq \sqrt{\frac{P_1}{2}} T_c$ and $c' \triangleq \frac{c}{\delta}$. Next (9.15) is substituted into (9.10), which is then solved for t_0. The Chernoff bound on the error probability of the soft-limiting correlation receiver is obtained via (9.11).

9.2. Asymptotic Analysis

In this section, we investigate the behavior of the soft-limiting DS/SSMA correlation receiver when the length of the signature sequences used per data bit becomes infinitely large. As mentioned earlier this analysis provides information about the limiting behavior of $\overline{P}_e$ that could be useful in cases when N is large but still finite. As before, we apply the central limit theorem and obtain a

Gaussian estimate of the error probability for the soft-limiting correlation receiver which is asymptotically exact.

Recall from the asymptotic analysis in Chapter 8 that, conditioned on the multiple-access parameters ρ_I, the test statistic is a sum of N independent random variables. Furthermore, note that the received data can be inflated by $\sqrt{N}$ to keep the correct scale, and then the test statistic is written as

$$Y_N = \sum_{j=0}^{N-1} \text{cl} \left[\tilde{\eta}_j + \sqrt{N} I_j'^{(1)} + \sqrt{\frac{P_1}{2N}} T b_0^{(1)} \right], \tag{9.16}$$

where $\tilde{\eta}_j$'s are samples with zero mean and variance $\frac{N_0 T}{4}$. In (9.16), $I_j'^{(1)}$'s are defined in (8.2) and (8.3). Applying the central limit theorem to the sum (9.16), the asymptotic bit-error probability for the soft-limiting DS/SSMA correlation receiver can be written as

$$\bar{P}_e = E_{\underline{\tau},\underline{\phi},\underline{b}} \left\{ Q \left[\frac{E_{\tilde{Y}}}{\sigma_{\tilde{Y}}} \right] \right\}, \tag{9.17}$$

where $E_{\tilde{Y}} \triangleq \lim_{N \to +\infty} E \left[\frac{Y_N}{\sqrt{N}} \mid b_0^{(1)} = +1, \rho_I \right]$ and $(\sigma_{\tilde{Y}})^2 \triangleq \lim_{N \to +\infty} \text{Var} \left[\frac{Y_N}{\sqrt{N}} \mid b_0^{(1)} = +1, \rho_I \right]$. Notice that (9.17) can be thought of as an estimate of the bit-error probability of the soft-limiting DS/SSMA correlation receiver which is asymptotically exact. We pursue the estimate based on (9.17) by elaborating on the argument of the Q function. For a given set of multiple-access parameters the normalized mean of the test statistic for the Laplacian channel example is given as

$$E_{\tilde{Y}} = \lim_{N\to+\infty} \left\{ \sqrt{\frac{P_1}{2}}T \left[1 + I_1(\underline{b},\underline{\tau},\underline{\phi}) - \frac{\sigma'}{\sqrt{2}} e^{-\frac{\sqrt{2}c'}{N\sigma'}} \sum_{j=0}^{N-1} \frac{\sinh[\sqrt{\frac{2}{N}}\frac{1}{\sigma'}(1+I_j^{(1)})]}{\sqrt{N}} \right] \right\}, \tag{9.18}$$

where $\sigma'^2 = \frac{N_0}{2E_b^{(1)}}$. Note that for very large N, (9.18) can be closely approximated by

$$E_Y \approx \sqrt{\frac{P_1}{2}}T \left[1 + I_1(\underline{b},\underline{\tau},\underline{\phi}) - e^{-\sqrt{\frac{8}{N_0T}}c} [1 + I_1(\underline{b},\underline{\tau},\underline{\phi}] \right], \tag{9.19}$$

where $Y \triangleq \frac{Y_N}{\sqrt{N}}$. Similarly, the variance of $\tilde{Y}$ is given as follows:

$$\sigma_{\tilde{Y}}^2 = \frac{N_0T}{4} + \lim_{N\to+\infty} \left\{ \frac{TE_b^{(1)}}{2N^2} \sum_{j=0}^{N-1} (1+I_j^{(1)})^2 - [\sqrt{\frac{N_0T}{2}}c + \frac{N_0T}{4}] e^{-\sqrt{\frac{8}{N_0T}}c} \right.$$

$$\left. \sum_{j=0}^{N-1} \frac{\cosh[\sqrt{\frac{4E_b^{(1)}}{N_0N}}(1+I_j^{(1)})]}{N} \right\} - \lim_{N\to+\infty} \frac{1}{N} \sum_{j=0}^{N-1} E^2\{\tilde{Y}_j \mid b_0^{(1)}=+1, \rho_I\}, \tag{9.20}$$

where $\tilde{Y}_j$ is the argument of cl in (9.16). Also notice that for large N, (9.20) can be approximated by

$$\sigma_{\tilde{Y}}^2 \approx \frac{N_0T}{4} - \left[\sqrt{\frac{N_0T}{2}}c + \frac{N_0T}{4} \right] e^{-\sqrt{\frac{8}{N_0T}}c}. \tag{9.21}$$

To obtain an estimate for the error probability of the soft-limiting correlation receiver, (9.19) and

(9.21) are substituted into (9.17) which yields

$$\overline{P}_e \approx E_{\underline{\tau},\underline{\phi},\underline{b}} \left\{ Q \left[\frac{\sqrt{\frac{2E_b^{(1)}}{N_0}} [1 + I_1(\underline{b},\underline{\tau},\underline{\phi})] (1 - e^{-\sqrt{\frac{8}{N_0 T}} c})}{\left[1 - (1 + \sqrt{\frac{8}{N_0 T}} c) e^{-\sqrt{\frac{8}{N_0 T}} c} \right]^{\frac{1}{2}}} \right] \right\}. \tag{9.22}$$

Note that the accuracy of the approximation (9.22) improves with N. It is also interesting to observe that the soft-limiter analysis in this chapter reduces to the analysis for the linear correlation receiver if we let $c = \infty$. On the other, hand notice that when the clipping level is set to zero, the receiver generates zero test statistic, and the resulting error probability is one half. Since the error probability of the soft-limiting correlation receiver is estimated by a simple form in (9.22), this expression can be used to elaborate on the performance of the receiver. In particular, (9.22) is compared with the expression for the error probability of the linear correlation receiver given in (6.1). In that analysis, if the distribution of the sum of N independent identically distributed random variables $\sum_{j=0}^{N-1} \eta'_j$ was approximated by the Gaussian distribution, the expression (6.1) would be estimated by

$$\overline{P}_e \approx E_{\underline{\tau},\underline{\phi},\underline{b}} \left\{ Q \left[\sqrt{\frac{2E_b^{(1)}}{N_0}} [1 + I_1(\underline{\tau},\underline{\phi},\underline{b})] \right] \right\} . \tag{9.23}$$

Note that for large enough N, (9.23) results in a good estimate of the error probability for the linear correlation receiver in Laplacian noise. Now comparing the approximation for the performance of the soft-limiting correlation receiver with that for the linear correlation receiver we notice that they differ by a factor in the argument of the Q function. From (9.22) this factor is seen as a function of the clipping level c and is given as

$$y(c) = \frac{1 - e^{-\sqrt{\frac{8}{N_0T}}c}}{\left[1-(1+\sqrt{\frac{8}{N_0T}}c)\, e^{-\sqrt{\frac{8}{N_0T}}c}\right]^{\frac{1}{2}}}. \tag{9.24}$$

Looking closely at (9.24), it can be shown that (9.24) is greater than unity for all finite values of clipping level c. Therefore, according to this application, soft-limiting correlation receiver outperforms the linear correlation receiver for the Laplacian channel example. The approximate performance of the soft-limiting correlation receiver can be evaluated via (9.22) by adjusting the signal-to-noise ratio and applying the technique developed in Chapter 6 for the linear correlation receiver. Particularly, the adjustment increases the effective signal-to-noise ratio in the following way:

$$\alpha_c = \alpha \frac{1 - e^{-\frac{\sqrt{2}\alpha c'}{N}}}{\left[1-(1+\frac{\sqrt{2}\alpha c'}{N})\, e^{-\frac{\sqrt{2}\alpha c'}{N}}\right]^{\frac{1}{2}}}, \tag{9.25}$$

where $\alpha \triangleq \sqrt{\frac{2E_b^{(1)}}{N_0}}$.

9.3. Numerical Results

The numerical results presented here are aimed at showing an improvement in performance by using soft-limiting correlation receivers in place of linear or hard-limiting correlation receivers when the channel is impulsive but not excessively so (e.g., Laplacian channel). However, due to the complexity of analyzing the performance of nonlinear DS/SSMA correlation receivers, we can only compare the exact error probabilities of linear and hard-limiting correlation receivers with the estimates of the error probability of the soft-limiting correlation receiver developed in the previous two sections.

The Chernoff upper bound is obtained by first substituting (9.15) into (9.10) and then solving for t_0, and the bound is then obtained via (9.11). Table 9.1 contains results for the Laplacian channel example with signal-to-noise ratios 8 and 4. These results are obtained with the same DS/SSMA system parameters considered in the examples in Chapter 5. These examples are carried out with lengths of signature sequences equal to 31, 63, 127, and 255 and different values of the clipping level c. Comparing Table 9.1 with Table 7.3 the Chernoff upper bound on the error probability of the soft-limiting correlation receiver results in lower upper bounds than that for the hard-limiting correlation receiver for all the values of N.

Again for the Laplacian channel example, we examine the Gaussian approximation introduced in the previous section which estimates the performance of the soft-limiting correlation receiver via (9.22) using the Taylor series expansion technique. These results which are asymptotically exact as $N \to +\infty$, are used in comparing soft-limiting correlation receivers with linear receivers and are depicted in Tables 9.2 and 9.3. These tables are generated with signal-to-noise ratio 8 and 4, respectively, and with different clipping level. The examples are carried out with lengths of signature sequences equal to 31, 63, 127, and 255. Here, by comparing the approximate error probabilities in Table 9.2 with the exact ones in Table 4.1, considerable improvement in performance is observed by using soft-limiting correlation receivers in place of linear correlators for the Laplacian channel example. However, note that the average error probability for the linear receiver is an exact computation, whereas results for the soft-limiting correlation receivers are merely approximations of the average error probability and are only asymptotically exact. Although these finding are not decisive they do provide a trend for further investigation on the subject.

TABLE 9.1. CHERNOFF UPPER BOUND FOR THE ERROR PROBABILITY OF SOFT-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; LAPLACIAN CHANNEL.

N	NORMALIZED CLIPPING LEVEL c'	K=1	K=2
	$SNR = 8\ dB$		
31	1.7	4.74×10^{-4}	2.94×10^{-3}
63	1.5	1.55×10^{-4}	2.57×10^{-4}
127	1.5	6.67×10^{-5}	1.24×10^{-4}
255	1.4	3.28×10^{-5}	5.01×10^{-5}
	$SNR = 4\ dB$		
31	1.4	2.67×10^{-2}	4.89×10^{-2}
63	1.4	1.94×10^{-2}	2.36×10^{-2}
127	1.4	1.47×10^{-2}	1.80×10^{-2}
255	1.5	1.21×10^{-2}	1.39×10^{-2}

TABLE 9.2. ASYMPTOTIC APPROXIMATION FOR ERROR PROBABILITY OF SOFT-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; LAPLACIAN CHANNEL, SNR=8.0 dB.

N	NORMALIZED CLIPPING LEVEL c'	K=1	K=2
31	1.0	4.95×10^{-7}	5.86×10^{-6}
	0.0521	2.60×10^{-7}	3.83×10^{-6}
63	1.0	3.56×10^{-7}	2.39×10^{-6}
	0.09	2.53×10^{-7}	1.89×10^{-6}
127	0.1	2.60×10^{-7}	6.51×10^{-7}
	0.148	2.52×10^{-7}	6.35×10^{-7}
255	0.1	4.52×10^{-7}	6.88×10^{-7}
	0.46	2.65×10^{-7}	4.19×10^{-7}

TABLE 9.3. ASYMPTOTIC APPROXIMATION FOR ERROR PROBABILITY OF SOFT-LIMITING CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; LAPLACIAN CHANNEL, SNR=4.0 dB.

N	NORMALIZED CLIPPING LEVEL c'	K=1	K=2
31	1.0	9.13×10^{-4}	1.47×10^{-3}
	0.1	7.72×10^{-4}	1.28×10^{-3}
63	1.0	8.35×10^{-4}	1.11×10^{-3}
	0.1001	7.49×10^{-4}	1.01×10^{-3}
127	1.0	7.98×10^{-4}	9.23×10^{-4}
	0.1	7.53×10^{-4}	8.73×10^{-4}
255	0.1	1.62×10^{-3}	1.70×10^{-3}
	0.81	7.73×10^{-4}	8.29×10^{-4}

CHAPTER 10

SUMMARY AND CONCLUSIONS

In this thesis we have considered an asynchronous multi-user communication problem over impulsive non-Gaussian channels. To have multiple-access capability the direct-sequence spread-spectrum technique was used. In the receiving end, linear and nonlinear correlation receivers were used and the performances of these receivers were evaluated by examining their bit-error probabilities. These nonlinear correlation receivers were developed by inserting a nonlinear element in the structure of the conventional linear correlator. This nonlinearity was introduced to limit the influence of the impulsive channel on the test statistic used by the receiver. A tractable way of studying correlation receivers in non-Gaussian impulsive channels was also introduced by modeling samples of noise after front-end filtering. The main assumption concerning the additive, zero mean channel noise is that these samples taken at the chip rate are independent. This allows us to study impulsive noise sources by modeling their post-sampling first-order probability distribution functions. Among various first-order non-Gaussian models we considered the ε-mixture of two Gaussian distributions, and the Laplacian distribution; these are tractable, commonly used empirical models for impulsive environments. The multi-user communication system under study was examined against these impulsive noise channels.

The model of the asynchronous binary PSK direct-sequence SSMA system considered here allowed a number, K, of users to share a channel. The linear and nonlinear correlators were assumed to be matched to the first of the users' signal. The structure of the nonlinear correlation receiver included an integrator, and the output was sampled at the chip rate. These samples were then passed through a memoryless nonlinearity. The decision on the parity of a data bit was based on the sum of N samples corresponding to that bit taken at the output of the nonlinear element. Among nonlinearities we were primarily interested in the hard-limiter, which is known to be very effective against impulsive disturbances in single-user communication problems. In particular, we compared the performance of the conventional linear correlator with that of the hard-limiter correlation receiver in impulsive and multi-user noises. Then we briefly examined the soft-limiting correlation receiver in a similar environment.

To examine the performance of linear correlation receivers we computed the average bit-error probability when multi-user and impulsive channel noises are interfering with a binary PSK signal. In all of our examples, a maximal-length spreading sequence of period N was assigned to each of the K users to reduce multiple-access interference. The principal conclusion here was that, with a fixed SNR, the linear correlation receiver does not perform as well as the Gaussian model predicts when the non-Gaussian noise has an impulsive nature (heavy-tailed distribution). In one case, in which the Gaussian assumption was violated in favor of an ε-mixture assumption, the resulting degradation in performance was equivalent to that caused by subscribing four additional users to a Gaussian channel with same SNR.

Motivated by the poor performance of the linear correlation against impulsive noise channels, we studied the performance of the hard-limiting correlation receiver under the same conditions as those for which the linear correlation receiver was analyzed. We first showed that hard-limiting correlation receivers offer significant improvement in performance over the linear correlation receiver in single-user impulsive channels. Then the combined effects of impulsive noise and multi-user interference on the performance of a hard-limiting correlation receiver were compared with those on the performance of a linear correlation receiver. These results indicated significant improvement in performance by using hard-limiting correlation receivers in place of linear correlators for more impulsive noise channels. Moreover, the improvement became more visible as the length of the signature sequences used by the channel subscribers increased. We also noted that, unlike the linear correlation receiver, degradation in performance due to the interfering user was no longer uniform in a range of signal-to-noise values. In fact, as the SNR increases, the hard-limiter apparently is not as effective in separating the two users as the linear correlation is. However, this deficiency is outweighed by the improvement against the impulsive noise as long as the channel noise is significant.

In the analysis of hard-limiting correlation receivers we observed that in examples of moderately impulsive channel noise with small N and K=2 the conventional linear receiver outperforms the hard-limiting correlation receiver. Comparing the two receivers, the linear correlator is more effective against multiple-access interference, whereas the hard-limiting correlation receiver is more effective against impulsive channel noise. We combined these desirable features and introduced soft-limiting correlation receivers as an alternative for moderately impulsive channels. Due to the complexity of analyzing the performance of soft-limiting correlation receivers, we could only compare the exact error probabilities of linear and hard-limiting correlation receivers with the estimates of the error probability of the soft-limiting correlation receiver. For the Laplacian channel example the approximate performance, which is asymptotically exact as $N \to +\infty$, indicates considerable improvement in performance by using the soft-limiting correlation receiver in place of both the linear and hard-limiting correlation receivers. Although these findings are not conclusive, they do provide a trend for further investigation on the subject.

To gain further insight into nonlinear correlation receivers and explore the fundamental limitation of these systems, we carried out an asymptotic analysis for linear and general nonlinear DS/SSMA correlation receivers. In this asymptotic analysis the length of the data period, the energy per data bit for a user, the noise power, and in turn the signal-to-noise ratio are kept constant. However, the length of the signature sequences used per data bit was allowed to grow without bound, and in turn the sampling period decreased to zero. Assuming some regularities on the nonlinearity, conditions were obtained under which a multi-user system achieves asymptotic single-user error probabilities in an infinite-user environment. Similar ideal asymptotic conditions on the MA signals were obtained when linear receivers were considered. An expression was also given for the asymptotic error probability of linear and nonlinear correlation receivers in Gaussian and non-Gaussian channels. This limiting behavior of the probability of error reinforced the idea that the performance of the multi-user communication system operating in impulsive environments can be improved significantly by taking into account the statistical characteristics of the channel noise.

APPENDIX A

NON-GAUSSIAN CHANNEL MODELS

In this Appendix we introduce some classes of first-order probability distribution functions that are widely used in modeling channel noise sources. These classes complement the ε-mixture and Laplacian noise models discussed in Chapter 2.

Among the physical models for impulsive noise, some of the most general are those developed by Middleton [19-20]. Canonical, statistical-physical models have been constructed and experimentally verified for a broad class of impulsive (mostly man-made) electromagnetic interference. In addition to avoiding the limitations of empirical models, these models also remain tractable for most analyses. Of particular interest to us are density functions in the Middleton Class A impulsive noise model which can be written as

$$f_{\eta_i}(x) = \sum_{m=0}^{\infty} K_m \mathcal{N}(0;C_m^2), \tag{A.1}$$

where $\mathcal{N}(0;C_m^2)$ denotes the zero mean Gaussian probability density function with variance C_m^2. Here $K_m = \dfrac{e^{-A}A^m}{m!}$ and $C_m^2 = \Omega_{2A}[\dfrac{m}{A}+\Gamma']$ where A, Γ' and Ω_{2A} are three parameters of the model. The variance of a random variable with density given by (A.1) is

$$\sigma^2 = \sum_{m=0}^{\infty} K_m C_m^2 = \Omega_{2A}(1+\Gamma') \ . \tag{A.2}$$

With the variance of the random variable fixed, there are only two free parameters, usually taken to be A and Γ'. The first free parameter, A, is called the impulsive index and measures the amount of temporal overlap among the waveforms of the interfering signals (inside the receiver). A large value of A means considerable overlap with a corresponding approach to Gaussianity, while a small value of A means highly impulsive or structured interference. The other variable, Γ', is given by the ratio of the power in the Gaussian portion of the interference to the power in the impulsive component. Middleton has shown that, by adjusting the parameters A and Γ', the density given in (A.1) can be made to fit a great variety of non-Gaussian densities quite well [19-20], and [36-37]. The parameters A and Γ' are physically motivated, unlike the parameters in the ε-mixture model, and can be estimated from physical measurements. (This problem has been studied recently by Zabin [46].) Vastola [40] has shown that in many cases the infinite series in (A.1) can be closely approximated by only two terms. This yields a way of choosing parameters of the mixture directly from the physically motivated parameters of the Class A model. For the Gaussian-Gaussian mixture, this relation stands as

$$\gamma^2 = 1+\frac{1}{A\Gamma'}, \tag{A.3}$$

and

$$\varepsilon = \frac{K_1}{K_0+K_1} = \frac{A}{1+A}. \tag{A.4}$$

Therefore, when an ε–mixture distribution is used to model narrow band non-Gaussian noise, ε and γ^2 can be obtained from (A.3) and (A.4) knowing that techniques are available for determining A and Γ'.

Among useful empirical models, the Cauchy density function is an example of a very heavy-tailed distribution. The density is given by

$$f_{\eta_i}(x) = \frac{c/\pi}{c^2+x^2}, \tag{A.5}$$

where c is a parameter controlling the scale. Due to its extremely heavy tails, the Cauchy density function has infinite variance. However, by carefully choosing the parameter c, it can still be used as a model for comparison with random variables of finite variance. This is done by equating the area under a truncated Cauchy density function with that of a Gaussian density with the desired variance. That is, we can use the relationship

$$1-2Q(b) = \int_{-b\sigma}^{b\sigma} \frac{c/\pi}{c^2+x^2} dx, \tag{A.6}$$

where $Q(b) \triangleq \frac{1}{\sqrt{2\pi}} \int_{b}^{\infty} e^{-\frac{x^2}{2}} dx$ is one minus the standard error function, and σ^2 is the noise variance to be matched. From this relationship c is obtained as

$$c = \frac{b\sigma}{\tan[\frac{\pi}{2} - \pi Q(b)]}. \tag{A.7}$$

(In the sequel we will choose b=1 in this context.) The Cauchy density function with parameter c given in (A.7) enables us to realize limitations in performance of the linear DS/SSMA systems in the presence of noise sources exhibiting extremely large impulses.

The last class of distributions to be considered here is the generalized Gaussian class. In [16], density estimates for non-Gaussian processes in the ocean acoustic environment have been shown to agree closely with generalized Gaussian density for different exponential decay rates. This class has a symmetric unimodal density obtained by generalizing the Gaussian density to obtain a variable rate of exponential decay and is given by (see [3] and [22])

$$f_{\eta_j}(x) = \frac{c}{2\Gamma(\frac{1}{c})A(c)} \exp\{-(\frac{|x|}{A(c)})^c\}, \tag{A.8}$$

where $A(c) = [\frac{\sigma^2\Gamma(1/c)}{\Gamma(3/c)}]^{1/2}$, σ^2 is the variance, $\Gamma(.)$ is the gamma function, and c is a positive parameter controlling the rate of decay. Note that for c=2 this density reduces to the Gaussian density, whereas for c=1 it becomes the Laplacian (double-exponential) density.

The classes of density functions introduced above cover a wide range of practical models for non-Gaussian sources, and all are of interest in practice. We analyze the performance of the linear correlation receiver in these non-Gaussian channels in Appendix B.

APPENDIX B

PERFORMANCE ANALYSIS OF THE LINEAR CORRELATION RECEIVER IN NON-GAUSSIAN NOISE

In Chapter 3, we evaluated the single-user average bit-error probability of the linear correlation receiver in ε-mixture and Laplacian noise channels. Then in Chapter 4, we added multi-user noise to the scenario and examined the performance of the linear receiver in these multi-user impulsive channels. In Appendix A, we introduced some alternative models of non-Gaussian channels that were also of interest in practice. In this Appendix, we examine the degradation in performance of the linear correlation receiver due to the channel noise modeled as Middleton Class A, Cauchy, and generalized Gaussian noise in single and multi-user cases by computing the exact average bit-error probability.

Recall from Chapter 3 that when K = 1 the average error probability for the linear receiver is obtained via

$$\bar{P}_e = \frac{1}{2} - \pi^{-1} \int_0^{\infty} u^{-1} (\sin u)\, \Phi_2(u)\, du, \tag{B.1}$$

where $\Phi_2(u) \triangleq \left[E\{e^{iu\eta_0}\} \right]^N$ is the characteristic function of the sum of the N independent identically distributed random variables, $\eta_0, \eta_1, \ldots \eta_{N-1}$ with zero means and variances $\frac{N_0}{2NE_b^{(1)}}$. The error probability for the Middleton Class A is computed by first finding an expression for the characteristic function $\Phi_2(u)$. The characteristic function for the Middleton Class A can be written as

$$\Phi_2(u) = \left[\sum_{m=0}^{\infty} K_m e^{-\frac{u^2 C_m^2}{2}} \right]^N, \tag{B.2}$$

where K_m's and C_m's are defined in Appendix A. The single-user average error probability is obtained by substituting (B.2) into (B.1). The expression for the $\Phi_2(u)$ is more complicated in the generalized Gaussian example. In this case the probability density function given in (A.8), which has an exponential form, is expanded and then used to compute the characteristic function. The final expression is as follows

$$\Phi_2(u) = \left[\sum_{m=0}^{\infty} \frac{(-1)^m}{2m!} u^{2m} A(c)^{2m} \frac{\Gamma(\frac{2m+1}{c})}{\Gamma(\frac{1}{c})} \right]^N , \tag{B.3}$$

where A(c) is defined in (A.8), $\Gamma(\cdot)$ is the gamma function, and c is a positive parameter controlling the rate of decay. The error is then obtained by substituting (B.3) into (B.1).

For the Cauchy example, the characteristic function is as

$$\Phi_2(u) = e^{-cN|u|} \tag{B.4}$$

where c is obtained from (A.7). Again, substituting (B.4) into (B.1) yields the error probability for linear correlators in Cauchy noise, which can be written in a closed form as

$$\overline{P}_e = \frac{1}{2} - \pi^{-1}\tan^{-1}(1/cN) . \tag{B.5}$$

To demonstrate the single-user performance of linear correlation receivers, Figures B.1 and B.2 have been generated for these non-Gaussian channel examples. Comparing the non-Gaussian channel examples with the Gaussian one, these curves indicate a degradation in performance for all impulsive noise cases over the entire range of interest of signal-to-noise ratios, with fairly large degradation in some cases. This is not surprising since the linear correlation receiver is designed to operate against

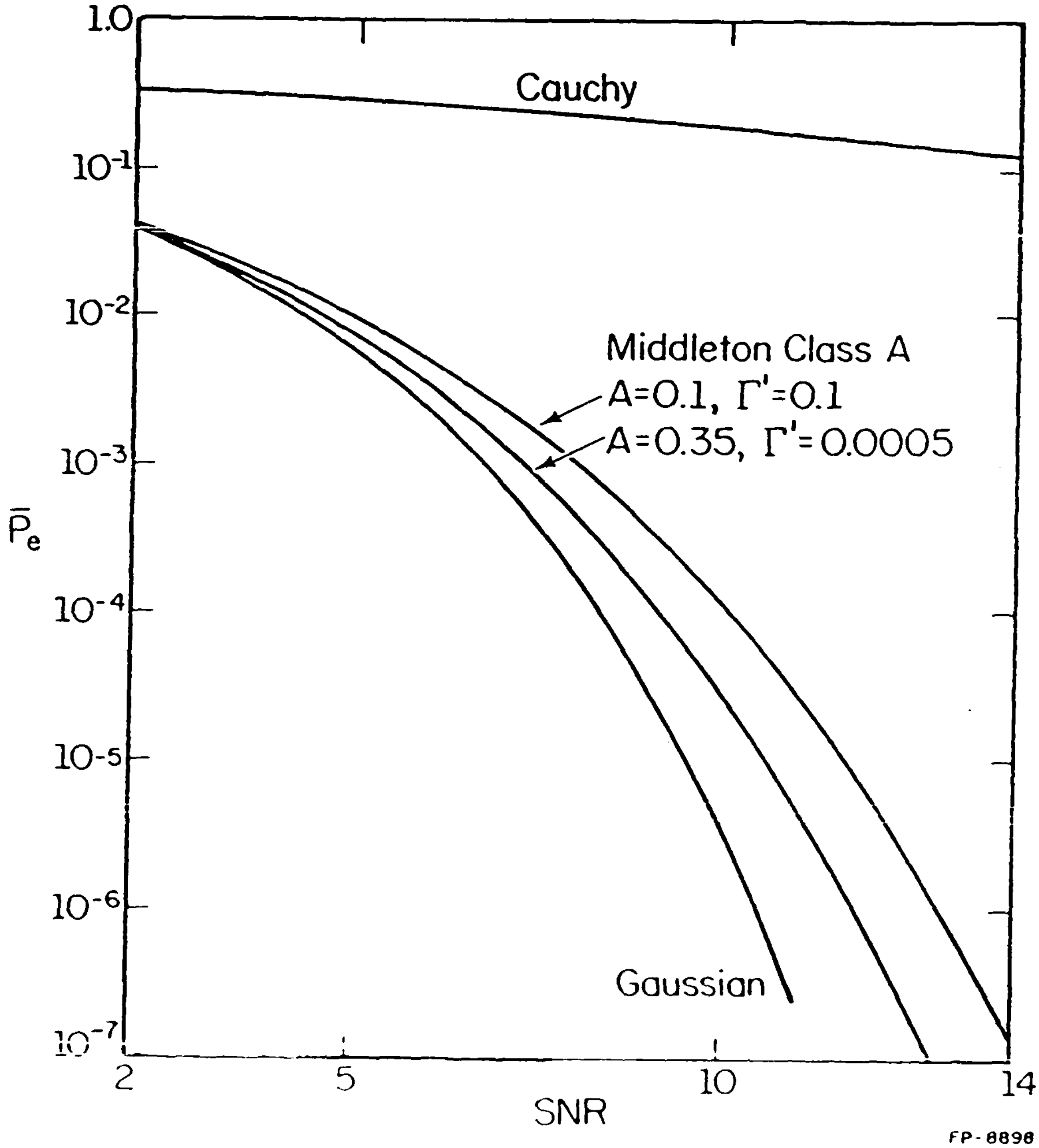

Figure B.1. Single-user error probability for the linear DS/SSMA correlation receiver in Gaussian, Cauchy, and Middleton Class A channels, N=31.

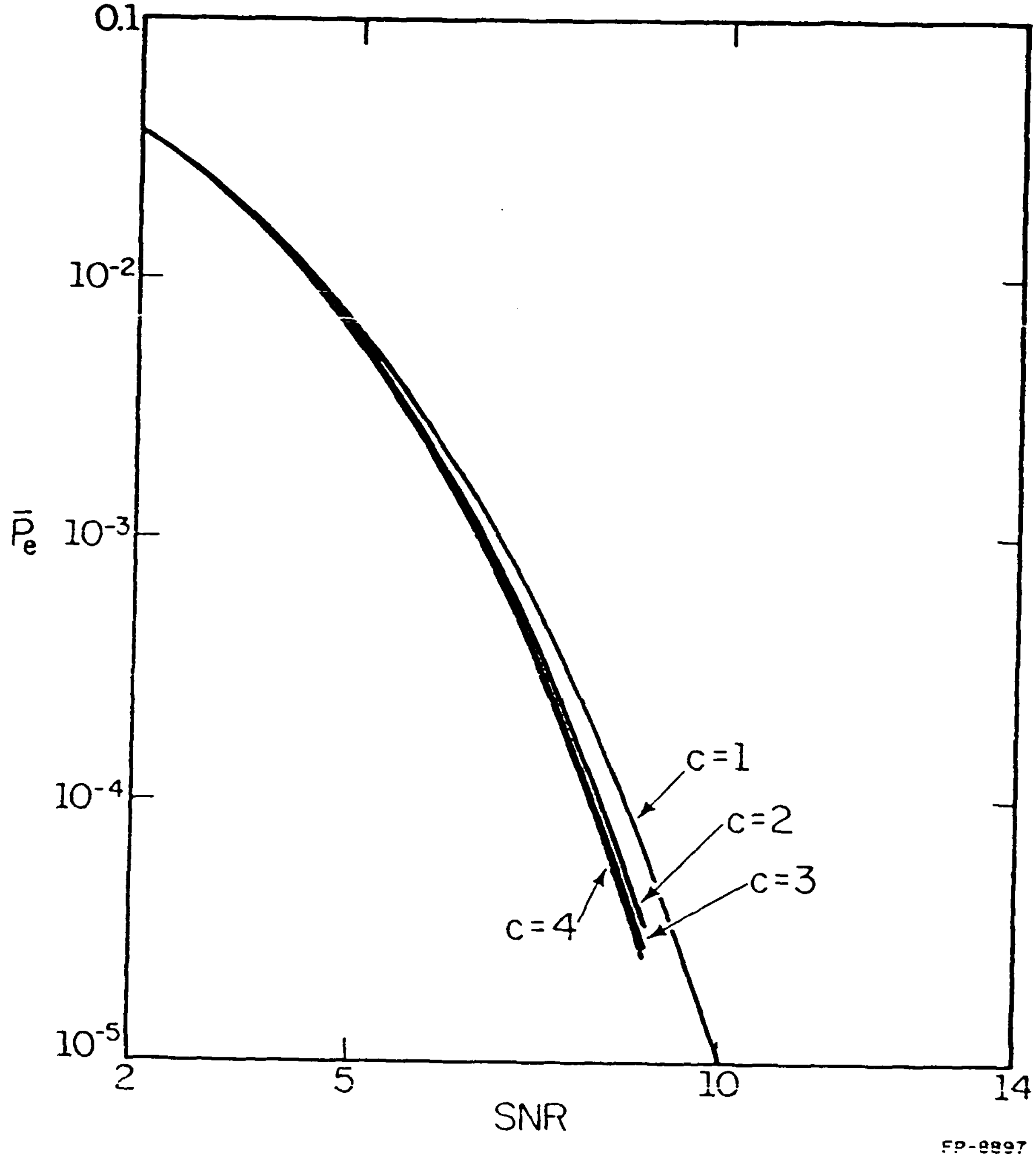

Figure B.2. Single-user error probability for the linear DS/SSMA correlation receiver in generalized Gaussian channels, N=31.

the Gaussian noise channel. Similar to our conclusion in Chapter 3 for the ε-mixture and Laplacian examples, these figures show that the impulsive character of the channel noise can drastically degrade the performance of conventional linear receivers.

Next we examine the performance of the linear receiver in the direct-sequence multi-user environment over Middleton Class A, generalized Gaussian, Cauchy noise channels. Recall from Chapter 4, that the multi-user bit-error probability can be written as

$$\overline{P}_e = \frac{1}{2} - \pi^{-1}\int_0^\infty u^{-1} (\sin u)\, \Phi_2 (u)\, du + \pi^{-1}\int_0^\infty u^{-1} (\sin u)\, \Phi_2 (u)\, [1-\Phi_1(u)]\, du, \tag{B.6}$$

where $\Phi_2 (u) \stackrel{\Delta}{=} [\, E\{ e^{iu\eta_0} \}\,]^N$ as in (B.1) and $\Phi_1 (u) \stackrel{\Delta}{=} E\{e^{iuI_1}\}$ with multiple-access interference I_1 given in (4.5). Since $\overline{P}_e$ is written conveniently in terms of characteristic functions, contributions of the MA interference and noise are distinguishable. Note that the left part of the expression for the multi-user error probability has already been evaluated for the non-Gaussian channels under study here. In particular, the characteristic functions $\Phi_2 (u)$ are given in (B.2), (B.3), and (B.4) for Middleton Class A, generalized Gaussian, and Cauchy channel examples, respectively. To evaluate the error probability via (B.6) we use the characteristic function of the multiple-access interference $\Phi_1 (u)$ given in (4.14). Therefore, the average bit-error probability of the linear correlation receiver for these non-Gaussian channels is obtained by substituting the corresponding expression for Φ_2 and (4.14) into (B.6). Tables B.1 contains the two-user error probability for the Middleton Class A and Cauchy channel examples with SNR = 8 dB and N = 31. Corresponding results for the generalized Gaussian channel example are depicted in Table B.2. For simplicity, the examples are carried through under the assumption that all users have the same power. All of these results are obtained with fixed signal-to-noise ratio, thereby showing the effects of the noise density shape on performance.

The principal conclusion here is that the linear receiver does not perform as well as the Gaussian model predicts when the non-Gaussian noise has an impulsive nature (heavy-tailed). For

TABLE B.1. AVERAGE ERROR PROBABILITY OF LINEAR CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; MIDDLETON CLASS A, CAUCHY AND GAUSSIAN CHANNELS, SNR=8.0 dB, N=31

DISTRIBUTIONS	K	P_e
MIDDLETON CLASS A		
$A = 0.0001, \Gamma' = 50.0$	1	4.59×10^{-4}
	2	6.72×10^{-4}
$A = 0.35, \Gamma' = 0.0005$	1	4.80×10^{-4}
	2	7.72×10^{-4}
$A = 0.1, \Gamma' = 0.1$	1	1.04×10^{-3}
	2	1.41×10^{-3}
	6	4.66×10^{-3}
GAUSSIAN	1	1.91×10^{-4}
	2	4.16×10^{-4}
	6	3.48×10^{-3}
CAUCHY	1	2.25×10^{-1}
	2	2.26×10^{-1}
	6	3.30×10^{-1}

TABLE B.2. AVERAGE ERROR PROBABILITY OF LINEAR CORRELATION RECEIVERS IN THE BINARY PSK DS/SSMA SYSTEM ; GENERALIZED GAUSSIAN CHANNELS, SNR=8.0 dB, N=31.

DECAY RATE c	K	P_e
1.0	1	2.94×10^{-4}
	2	5.45×10^{-4}
2.0	1	1.91×10^{-4}
	2	4.16×10^{-4}
3.0	1	1.72×10^{-4}
	2	3.91×10^{-4}
4.0	1	1.64×10^{-4}
	2	3.82×10^{-4}

comparison we have also shown short-tailed distributions in Table B.2 ($c = 3$ and $c = 4$), in which performance of the linear receiver is better than predicted. However, these short-tailed distributions do not model impulsive phenomena.

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